$\boxed{1A}$

$$\left|\frac{n+1}{3n-1} - \frac{1}{3}\right| = \left|\frac{\overset{\cancel{3n}+3}{3(n+1)}}{3(3n-1)} - \frac{\overset{-\cancel{3n}+1}{1(3n-1)}}{3(3n-1)}\right|$$

seq. values − Limit

$$= \left|\frac{4}{3(3n-1)}\right| = \frac{4}{9n-3} < \frac{1}{1000}$$

$$4000 < 9n-3$$

$$4003 < 9n$$

$$n > 444$$

Prove: $\lim\limits_{n\to\infty} \dfrac{n+1}{3n-1} = \dfrac{1}{3}$

Let $\varepsilon > 0$ be given.

$$\left|\frac{n+1}{3n-1} - \frac{1}{3}\right| = \frac{4}{9n-3} < \varepsilon \Rightarrow 4 < 9n\varepsilon - 3\varepsilon$$

$$\boxed{n > \frac{4+3\varepsilon}{9\varepsilon}}$$

$\boxed{\text{IB}}$

#2a Show that

$$\frac{3n-1}{4n-3} \le \frac{3n+\sin(2n)}{4n-3} \le \frac{3n+1}{4n-3}$$

$$\uparrow \qquad\qquad\qquad\qquad \uparrow$$

show $\cancel{3n}-1 \le \cancel{3n}+\sin(2n) \le \cancel{3n}+1$

show $-1 \le \sin(2n) \le 1$ ✓

2c

Since $\dfrac{3n-1}{4n-3} \le \dfrac{3n+\sin(2n)}{4n-3} \le \dfrac{3n+1}{4n-3}$

and $\lim\limits_{n\to\infty} \dfrac{3n-1}{4n-3} = \lim\limits_{n\to\infty} \dfrac{3n+1}{4n-3} = \dfrac{3}{4}$

then by the Squeeze Thm,

$$\lim_{n\to\infty} \frac{3n+\sin(2n)}{4n-3} = \frac{3}{4}$$

#3a $\frac{1}{2} \le \frac{2n}{3n+1}$ | $3\left(\frac{2n}{3n+1}\right) < \left(\frac{2}{\cancel{3}}\right)\cancel{3}$

$3n+1 \le 4n$ | $6n < 6n+2$

$1 \le n$ ✓ | $0 < 2$ ✓

Since $\lim_{n\to\infty}\left(\frac{1}{2}\right)^n = 0 = \lim_{n\to\infty}\left(\frac{2}{3}\right)^n$

and $\left(\frac{1}{2}\right)^n \le \left(\frac{2n}{3n+1}\right)^n < \left(\frac{2}{3}\right)^n$

Then $\lim_{n\to\infty}\left(\frac{2n}{3n+1}\right)^n = 0$

$\mathbb{N}$ = natural numbers $0, \underbrace{1, 2, 3, \ldots}_{\mathbb{N}^+}$

The integers

$\mathbb{Z} = \{\ldots -2, -1, 0, 1, 2, \ldots\}$

$\mathbb{Q}$ = the <u>rationals</u> (includes all integers, $\frac{2}{3}, -\frac{5}{8}, \ldots$

$\mathbb{R}$ – the reals

How do we know that

$$\lim_{n\to\infty} \frac{n^2}{n+1} = \infty$$

Because for any (huge) $L > 0$, you can go far enough out in the sequence so that <u>every</u> <u>term</u> is larger than L.

<u>Thm</u> If $\lim_{n\to\infty} |u_n| = \infty$

then $\lim_{n\to\infty} \frac{1}{u_n} = 0$

If $\lim_{n\to\infty} u_n = 0$

then $\lim_{n\to\infty} \left|\frac{1}{u_n}\right| = \infty$

$\times$ Definition

$f: I \to \mathbb{R}$ for every number in the open interval I, f(that number) is a real number

function $\quad (-1, .5)$

$a \in I$

"is an element of"

"belongs to"

Limits of Function

$$\boxed{\lim_{x \to a} f(x) = b \iff \text{for } \underline{\text{any}} \ \underline{\text{sequence}} \ \{a_n\} \text{ such that } \lim_{n \to \infty} a_n = a, \ \lim_{n \to a} [f(a_n)] = b}$$

Ex. show that $\lim_{x \to 2} x^2 = 4$

$\{1.9, 1.99, 1.999, \ldots\} \longrightarrow 2$

$\{f(1.9), f(1.99), f(1.999), \ldots\} \to 4$

$\{3.61, 3.9601, 3.996001, \ldots\}$

If $f(\{u_n\}) = \{u_n^2\} \to 4$ for any

sequence

sequence where $\lim_{n\to\infty} u_n = 2$, then $\lim_{x\to 2} x^2 = 4$.

Useful Observation

To show $\lim_{x\to a} f(x)$ dne

Find 2 sequences that both go to a

$\left[\text{say, } \lim_{n\to\infty} u_n = a \text{ and } \lim_{n\to\infty} v_n = a\right]$

and show that $\lim_{n\to\infty} f(u_n) \neq \lim_{n\to\infty} f(v_n)$