What is a limit (really)?

Ex. Show that $\lim_{n \to \infty} \frac{4n+1}{n+2} = 4$

Can you go out far enough in the sequence so that every term thereafter is within 0.001 of 4?

$$\left| \underbrace{\frac{4n+1}{n+2}}_{\text{terms of the sequence}} - \underbrace{4}_{\text{Limit}} \right| = \left| \frac{\cancel{4n}+1}{n+2} - \frac{4(\cancel{n}+2)}{n+2} \right|$$

$$= \left| \frac{-7}{n+2} \right| = \frac{7}{n+2} < 0.001$$

$$\frac{7}{n+2} < \frac{1}{1000}$$

$$7000 < n+2$$

$$6998 < n$$

$$\boxed{n > 6998}$$

check

$$\frac{4(6999)+1}{6999+2}$$

Ex. Show $\lim_{n \to \infty} \frac{2n+4}{3n+1} = \frac{2}{3}$

Let $\varepsilon = 0.0001$

epsilon
(small positive value)

$$\left| \frac{2n+4}{3n+1} - \frac{2}{3} \right| = \left| \frac{(2n+4)3}{(3n+1)3} - \frac{2(3n+1)}{3(3n+1)} \right|$$

$$= \left| \frac{\cancel{6n}+12-\cancel{6n}-2}{3(3n+1)} \right| = \frac{10}{3(3n+1)} < \frac{1}{10000}$$

$100000 < 9n+3$

$99997 < 9n$

$n > 11110$

check

$$\frac{2(11111)+4}{3(11111)+1}$$

$p \leftrightarrow q$

Defn $\lim_{n\to\infty} u_n = L$ iff (if and only if) for any $\varepsilon > 0$ there exists M such that if $n > M$ then $|u_n - L| < \varepsilon$.

Ex. Prove: $\lim_{n\to\infty} \frac{7n+7}{n-1} = 7$, $n > 1$

Let $\varepsilon > 0$ be given.

$$\left| \frac{7n+7}{n-1} - 7 \right| = \left| \frac{7n+7}{n-1} - \frac{7(n-1)}{n-1} \right|$$

$\cancel{7n} + 7 \quad - \cancel{7n} + 7$

$$= \frac{14}{n-1} < \varepsilon$$

$$14 < \varepsilon n - \varepsilon$$

$$14 + \varepsilon < \varepsilon n$$

$$\boxed{n > \frac{14+\varepsilon}{\varepsilon}}$$

Subsequences

Ex. $0, 1, 0, 1, 0, 1, \ldots$ ← diverges

$\{u_{2n-1}\}$ converges to 0

subsequence

$\{u_{2n}\}$ converges to 1

Thm If $\{a_n\}$ and $\{b_n\}$ are subsequences of $\{u_n\}$ and $\lim_{n\to\infty} a_n \neq \lim_{n\to\infty} b_n$ then $\{u_n\}$ diverges

Thm If $\{u_n\}$ converges and $\{a_n\}$ and $\{b_n\}$ are subsequences of $\{u_n\}$, then $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = \lim_{n\to\infty} u_n$

The Squeeze Theorem

(pinching, sandwich)

If $\lim_{n\to\infty} a_n = L$ and $\lim_{n\to\infty} b_n = L$

and for all $n > M$ $a_n \le u_n \le b_n$

then $\lim_{n\to\infty} u_n = L$.

Prove: $\lim_{n\to\infty} \frac{\sin(2n+1)}{n} = 0$

$$\frac{-1}{n} \le \frac{\sin(2n+1)}{n} \le \frac{1}{n}$$

smallest sine gets

biggest sine value

$$\lim_{n\to\infty} \frac{-1}{n} = 0 = \lim_{n\to\infty} \frac{1}{n}$$

so $\lim_{n\to\infty} \frac{\sin(2n+1)}{n} = 0$

HW <u>1A</u> #2

<u>1B</u> #2, 3