

What are limits? (Really)

$$\text{Ex. } \lim_{n \rightarrow \infty} \frac{2n+1}{3n+2} = \frac{2}{3}$$

Can we go far enough out in the sequence so that every term is within 0.01 of $\frac{2}{3}$?

$$\left| \frac{2n+1}{3n+2} - \frac{2}{3} \right| = \left| \frac{3(2n+1)}{3(3n+2)} - \frac{2(3n+2)}{3(3n+2)} \right|$$

$$= \left| \frac{\cancel{6n}+3-\cancel{6n}-4}{9n+6} \right| = \frac{1}{9n+6} < 0.01$$

$$9n+6 > 100 \Rightarrow n > \frac{94}{9} = 10.4$$

$$u_{11} = \frac{2(11)+1}{3(11)+2} = 0.65714$$

$$0.65714 - \frac{2}{3} = -0.009$$