# L'Hôpital's Rule

#4. $\lim_{x\to 0} \dfrac{e^x - (1-x)}{x^3}$

$e^x - 1 + x$

$\frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to 0} \dfrac{e^x + 1}{3x^2} = \infty$

$\frac{2}{0}$

#6. $\lim_{x\to 0} \dfrac{\sin^{-1} x}{x}$

$\frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to 0} \dfrac{\frac{1}{\sqrt{1-x^2}}}{1}$

$= 1$

## Constant Multiple Rule

Ex $f(x) = \underline{10}\, e^x$

$f'(x) = 10 e^x$

## Indeterminate forms

- $\frac{0}{0}$
- $\frac{\infty}{\infty}$

$\lim_{x\to\infty} \dfrac{x^3}{e^x} = 0$

- $1^\infty$

(Take a log and exponentiate the answer)

- $0 \cdot \infty$

(Take one factor to the denom.)

- $\infty - \infty$

(usually - get a common denom <u>OR</u> mult. by the conjugate)

- $0^0$

#7. $$\lim_{x \to 1} \frac{\tan^{-1} x - \frac{\pi}{4}}{x-1}$$

$\frac{0}{0}$

$$\overset{L'H}{=} \lim_{x \to 1} \frac{\frac{1}{1+x^2}}{1}$$

$$= \frac{1}{2}$$

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#9 $$\lim_{x \to \infty} \left( x \cdot \sin \frac{1}{x} \right) =$$

$\infty \cdot 0$

$$= \lim_{x \to \infty} \frac{\sin \frac{1}{x}}{\frac{1}{x}}$$

$\frac{0}{0}$

$$\overset{L'H}{=} \lim_{x \to \infty} \frac{\cos\left(\frac{1}{x}\right) \cdot \cancel{\frac{-1}{x^2}}}{\cancel{-\frac{1}{x^2}}} = 1$$

$\underline{\#14}$ $\lim_{x\to\infty}\left(1-\frac{2}{x}\right)^x = e^{-2} = \frac{1}{e^2}$

$1^\infty$

$\underline{\text{New problem}}$ $\lim_{x\to\infty} \ln\left(1-\frac{2}{x}\right)^x$

$a\ln b = \ln b^a$

$= \lim_{x\to\infty} \frac{\cancel{x}\ln\left(1-\frac{2}{x}\right)}{\frac{1}{x}} \overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{1-2/x}\cdot\frac{+2}{\cancel{x^2}}}{-\frac{1}{\cancel{x^2}}}$

$\frac{0}{0}$

$1$

$= -2$

$-2x^{-1}$

$+2x^{-2}$

#19 $\lim_{x\to 1}\left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$

$\infty - \infty$

$= \lim_{x\to 1}\left(\frac{1(x-1)}{\ln x\,(x-1)} - \frac{1\cdot \ln x}{(x-1)\ln x}\right)$

$\frac{0}{0}$ $= \lim_{x\to 1}\left(\frac{x-1-\ln x}{(x-1)\cdot \ln x}\right)$

$\overset{L'H}{=} \lim_{x\to 1}\left(\frac{1-\frac{1}{x}}{(x-1)\cdot\frac{1}{x}+\ln x}\right)$

$\frac{0}{0}$

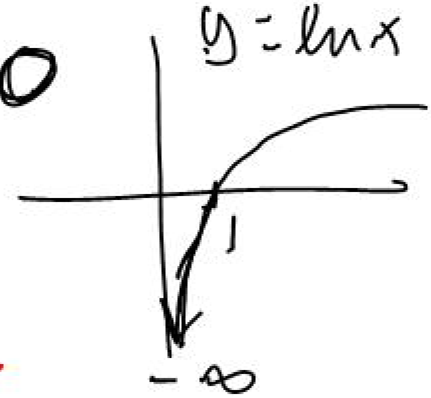
$= \lim_{x\to 1}\frac{\frac{x-1}{\cancel{x}}}{\frac{x-1+x\ln x}{\cancel{x}}}$

$\frac{0}{0}$ $= \lim_{x\to 1}\frac{x-1}{x-1+x\ln x}$

$\overset{L'H}{=} \lim_{x\to 1}\frac{1}{1+1+\ln x} = \frac{1}{2}$

Ex $\lim\limits_{x\to 0^+} (\tan x)^{\sin x} = 0$

$0^0$




New prob

$$\lim_{x\to 0^+} \frac{\cancel{\sin x}\,\ln(\tan x)^{\cancel{\sin x}}}{\csc x}$$

$\frac{-\infty}{\infty}$

NOTE

$\frac{1}{\sin x} = \csc x$

$$\overset{LH}{=} \lim_{x\to 0^+} \frac{\frac{\sec^2 x}{\tan x}}{-\csc x \cot x}$$

$$= \lim_{x\to 0^+} \frac{\cancel{\cos x}}{\cancel{\cos^2 x}\sin x} \cdot \frac{-\cancel{\cos x}}{\sin x \sin x}$$

$$= \lim_{x\to 0^+} \frac{-1}{\sin^3 x} = -\infty$$

$$e^{-\infty} = \frac{1}{e^{\infty}} \to 0$$

# L'Hôpital's Rule

- If $\lim_{x \to a} f(x) = 0$ and

  $\lim_{x \to a} g(x) = 0$

  then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$.

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- If $\lim_{x \to a} f(x) = \infty$ and

  $\lim_{x \to a} g(x) = \infty$

  then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

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$$\frac{1}{1/10} = 10$$

$$\frac{1}{\frac{1}{1000}} = 1000$$

$$\lim_{x \to \infty} \frac{\frac{1}{x+2}}{\frac{3x}{x-4}}$$

HW # 12, 15, 20, 23, 24

Quiz next class