L'Hôpital problems

#12 $\lim_{x\to 0^+} (e^x + x)^{\frac{1}{x}} = e^2$

$\boxed{1^\infty}$

exponentiate

New problem

$$\lim_{x\to 0^+} \frac{\ln(e^x+x)}{x} \overset{L'H}{=} \lim_{x\to 0^+} \frac{\frac{e^x+1}{e^x+x}}{1} = 2$$

$\boxed{\frac{0}{0}}$

#15 $\lim_{x\to 0^+} (1-x)^{\frac{1}{x}} = e^{-1} = \frac{1}{e}$

$\boxed{1^\infty}$

exponentiate

New problem

$$\lim_{x\to 0^+} \frac{\ln(1-x)}{x} \overset{L'H}{=} \lim_{x\to 0^+} \frac{\frac{-1}{1-x}}{1} = -1$$

$\boxed{\frac{0}{0}}$

#20 $\lim_{x\to\infty} (x - \ln x) \cdot \frac{x}{x} = \lim_{x\to\infty} x \cdot \left(\frac{x}{x} - \frac{\ln x}{x}\right)$

$\boxed{\infty - \infty}$

$$= \lim_{x\to\infty} x\left(1 - \frac{\ln x}{x}\right) = \infty$$

$\to \infty(1-0)$

what happens to this term? $\lim_{x\to\infty} \frac{\ln x}{x} \overset{L'H}{=} \frac{1/x}{1}$

$\frac{\infty}{\infty}$

$= 0$

$\boxed{\#23}$ $\lim_{x\to 0^+} (\tan 2x)^x = e^0$ ← exponentiate

$\boxed{0^0}$ $= \boxed{1}$

New problem

$$\lim_{x\to 0^+} \frac{\ln(\tan 2x)^{\cancel{x}}}{\frac{1}{x}} \overset{L'H}{=} \lim_{x\to 0^+} \frac{\frac{2\cdot\sec^2 2x}{\tan 2x}}{-\frac{1}{x^2}}$$

$\boxed{\frac{-\infty}{\infty}}$

$$= \lim_{x\to 0^+} \frac{-x^2\cdot \cancel{\cos x}\cdot 2}{\cos^{\cancel{2}} 2x \sin x}$$

$$= \lim_{x\to 0^+} \frac{-2x^2}{\sin x\cdot\cos x}$$

$\boxed{\frac{0}{0}}$

$$\overset{L'H}{=} \lim_{x\to 0^+} \frac{-4x}{\sin x\cdot(-\sin x) + \cos x\cdot\cos x}$$

$$= \lim_{x\to 0^+} \frac{-4x}{\cos^2 x - \sin^2 x}$$

$$= \lim_{x\to 0^+} \frac{-4x}{\cos 2x}$$

Double Angle identity for cosine

$$= 0$$

$\boxed{\#24}$ $\lim_{x\to\infty}\left(\frac{2x-3}{2x+5}\right)^{2x+1} = e^{-8} = \frac{1}{e^8}$

$\boxed{1^\infty}$

exponentiate

New problem

$$\lim_{x\to\infty} \ln\left(\frac{2x-3}{2x+5}\right)^{2x+1} = \lim_{x\to\infty} \frac{\ln\left(\frac{2x-3}{2x+5}\right)}{\frac{1}{2x+1}}$$

$\boxed{\frac{0}{0}}$

$$\stackrel{L'H}{=} \lim_{x\to\infty} \frac{\frac{2x+5}{2x-3}\cdot\frac{(2x+5)(2)-(2x-3)(2)}{(2x+5)^2}}{\frac{-2}{(2x+1)^2}}$$

$$= \lim_{x\to\infty} \frac{\frac{\cancel{2x+5}^{\,1}}{2x-3}\cdot\frac{\cancel{16}\,8}{(2x+5)^{\cancel{2}}}}{\frac{\cancel{-2}\,{-1}}{(2x+1)^2}}$$

$$= \lim_{x\to\infty} \frac{-8(2x+1)^2}{(2x-3)(2x+5)} = \frac{-32}{4} = -8$$