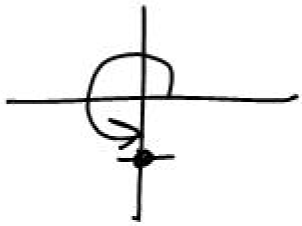
Complex Review

#1 (a) $\sqrt{3^2+(-4)^2} = 5$

(b) $\sqrt{0^2+(-10)^2} = 10$

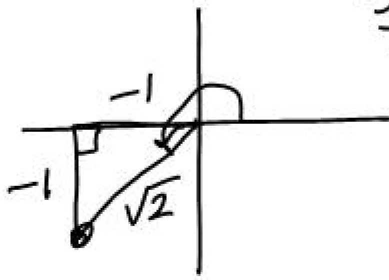
#2 (a)



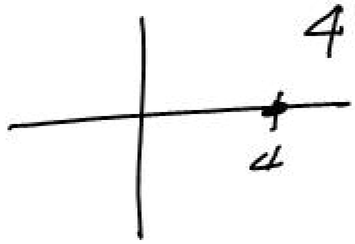
$\frac{3\pi}{2}$ or $-\frac{\pi}{2}$

(b)

$\frac{5\pi}{4}$ or $-\frac{3\pi}{4}$

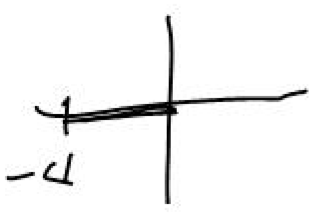


#3 (a)



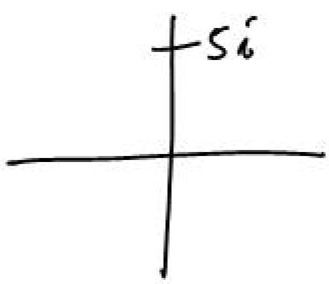
$4 = 4 \operatorname{cis} 0$ or $4 \operatorname{cis} 2\pi$

(b)



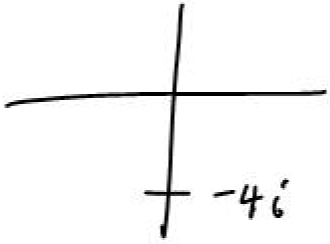
$-4 = 4 \operatorname{cis} \pi$ or $4 \operatorname{cis}(-\pi)$

(c)



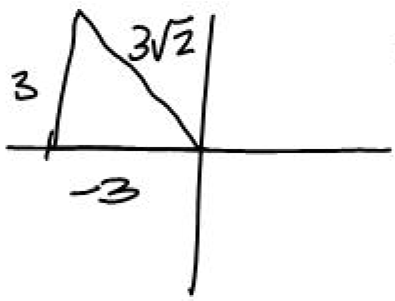
$5i = 5 \operatorname{cis} \frac{\pi}{2}$

(d)



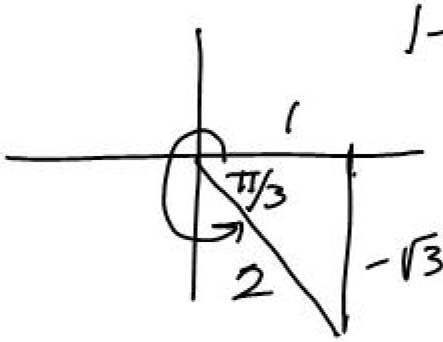
$-4i = 4 \text{ cis } \frac{3\pi}{2}$ or $4 \text{ cis } \frac{-\pi}{2}$

(e)



$-3+3i = 3\sqrt{2} \text{ cis } \frac{3\pi}{4}$

(f)



$1-\sqrt{3}i = 2 \text{ cis } \frac{5\pi}{3}$ or $2 \text{ cis } \left(-\frac{\pi}{3}\right)$

(4) (a) $\text{cis } 45° = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i$ or $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$

(b) $\text{cis } \frac{5\pi}{6} = -\frac{\sqrt{3}}{2} + \frac{1}{2} i$

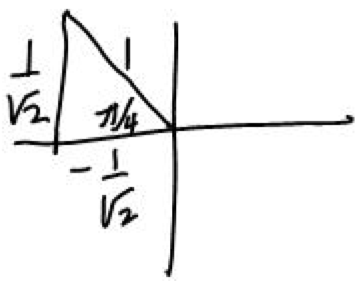
(c) $4 \text{ cis } 120° = 4\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) = -2 + 2\sqrt{3} i$

(d) $2 \text{ cis } \frac{3\pi}{4} = 2\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right) = -\sqrt{2} + \sqrt{2} i$

(5) $\left(\operatorname{cis} \frac{2\pi}{3}\right)^5 = \operatorname{cis} \frac{10\pi}{3} = \operatorname{cis} \frac{4\pi}{3}$

$= -\frac{1}{2} - \frac{\sqrt{3}}{2} i$

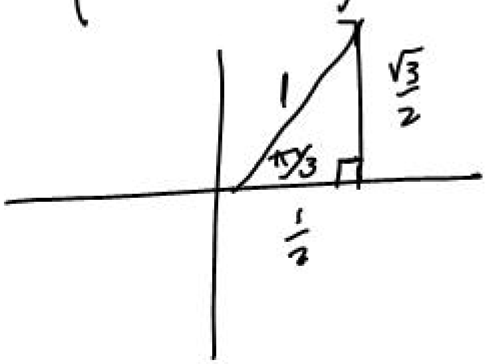
(6) $\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i\right)^4 = \left(\operatorname{cis} \frac{3\pi}{4}\right)^4 = \operatorname{cis} 3\pi$



$= \operatorname{cis} \pi$

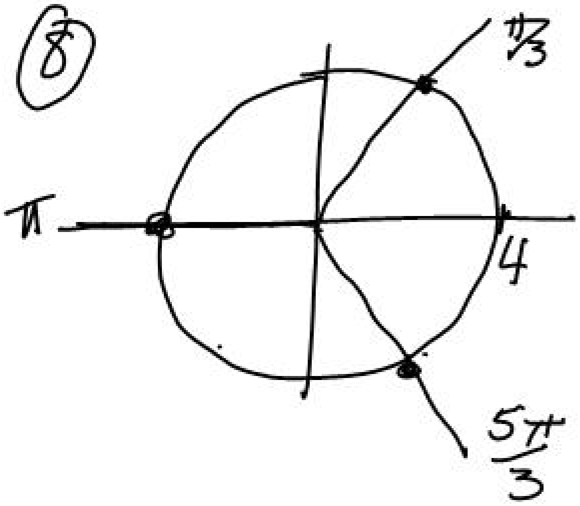
$= -1$

(7) $\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)^6 = \left(\operatorname{cis} \frac{\pi}{3}\right)^6 = \operatorname{cis} 2\pi$



$= 1$

(8)



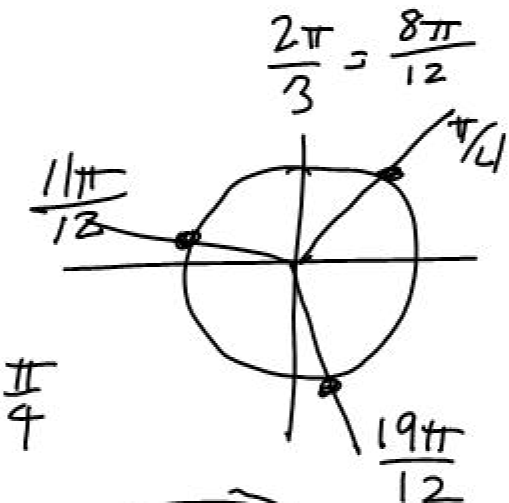
$\frac{2\pi}{3}$

- $4 \text{ cis } \pi = -4$
- $4 \text{ cis } \frac{5\pi}{3} = 4\left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)$

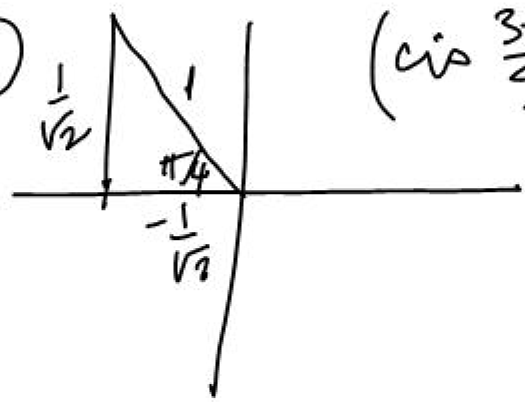
$= 2 - 2\sqrt{3}\, i$

(9) $\left(3 \text{ cis } \overset{\frac{4\pi}{12}}{\frac{\pi}{3}}\right)\left(4 \text{ cis } \overset{\frac{15\pi}{12}}{\frac{5\pi}{4}}\right) = 12 \text{ cis}\left(\frac{19\pi}{12}\right)$

(10) $\dfrac{9 \text{ cis } \overset{\frac{9\pi}{12}}{\frac{3\pi}{4}}}{3 \text{ cis } \underset{\frac{4\pi}{12}}{\frac{\pi}{3}}} = 3 \text{ cis } \frac{5\pi}{12}$



(11)



$\left(\text{cis } \frac{3\pi}{4}\right)^{\frac{1}{3}} = \text{cis } \frac{\pi}{4}$

over

$$\boxed{\begin{aligned}
&\bullet\ \operatorname{cis}\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \quad \text{or} \quad \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \\
&\bullet\ \operatorname{cis}\frac{11\pi}{12} = \frac{-\sqrt{2}-\sqrt{6}}{4} + \frac{\sqrt{6}-\sqrt{2}}{4}i \\
&\bullet\ \operatorname{cis}\frac{19\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4} - \frac{\sqrt{2}+\sqrt{6}}{4}i
\end{aligned}}$$

$$\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}+\sqrt{6}}{4}$$

$$\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos\frac{\pi}{3}$$

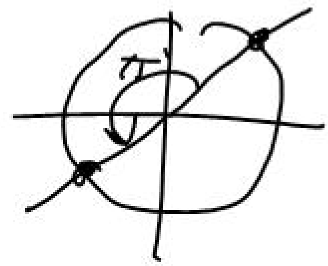
$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

- z^*

$z = 4 - 2i \quad , \quad z^* = 4 + 2i$

- $z = r \operatorname{cis} \theta$

$-z = r \operatorname{cis} (\theta + \pi)$



$z^* = r \operatorname{cis} (-\theta)$

$-z^* = r \, (\pi - \theta)$

$$\frac{1}{z} = \frac{1}{r} \operatorname{cis} (-\theta)$$

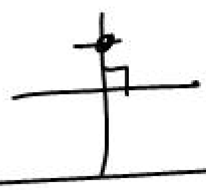
$$r \operatorname{cis} \theta \cdot s \operatorname{cis} \gamma = rs \operatorname{cis} (\theta + \gamma)$$

$$\frac{r \operatorname{cis} \theta}{s \operatorname{cis} \gamma} = \frac{r}{s} \operatorname{cis} (\theta - \gamma)$$

• $r \operatorname{cis}\theta = re^{i\theta}$ (Euler form)

$$i^i = \left(e^{i\frac{\pi}{2}}\right)^i = e^{i^2 \cdot \frac{\pi}{2}} = e^{-\frac{\pi}{2}} = \frac{1}{e^{\pi/2}}$$

$$i = \operatorname{cis}\frac{\pi}{2} = e^{i\frac{\pi}{2}}$$



(Real!)

• Solve Equations: $z^4 = -\sqrt{3} + i$

• Find roots of unity