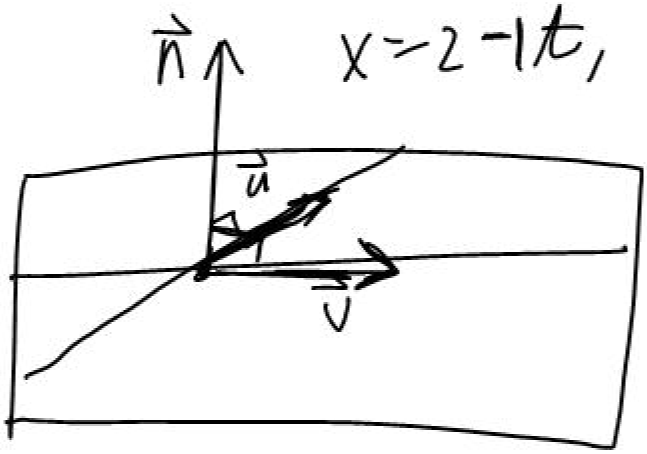
#3 c $\quad 1 - x = y - 1 = 2z \leftarrow$ Cartesian

$x = 2 - 1t, \; y = 1 + 2t, \; z = 1t \leftarrow$ parametr



$$\frac{x - x_0}{x\text{-comp}} = \frac{y - y_0}{y\text{-comp}} = \frac{z - z_0}{z\text{-comp}}$$

$$\frac{x - 1}{-1} = \frac{y - 1}{1} = \frac{z - 0}{1/2}$$

$$\vec{u} = \begin{pmatrix} -1 \\ 1 \\ 1/2 \end{pmatrix} \text{ or } \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$$

$$\vec{v} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

$$(2\vec{i} - \vec{j} - 4\vec{k}) - (-2\vec{k} + 2\vec{i} - 2\vec{j})$$

$$\vec{u} \times \vec{v} = \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 2 & 1 \\ -1 & 2 & 1 \end{pmatrix} \begin{matrix} \vec{i} & \vec{j} \\ -2 & 2 \\ -1 & 2 \end{matrix} = \vec{j} - 2\vec{k} = \vec{n}$$

point on plane : $(2, 1, 0)$

$y - 2z = \boxed{1} \leftarrow$ cartesian eq.

#3c** Find a vector eq. for the plane

$$\boxed{\vec{r} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}}$$

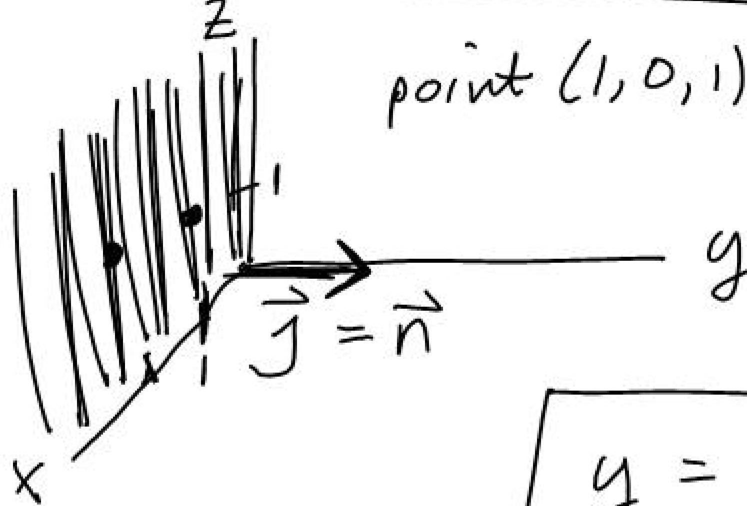
vectors in the plane

or

$$\boxed{\vec{r} = (2-2\alpha-\beta)\vec{\imath} + (1+2\alpha+2\beta)\vec{\jmath} + (\alpha+\beta)\vec{k}}$$

#4

point (1,0,1)



plane x y

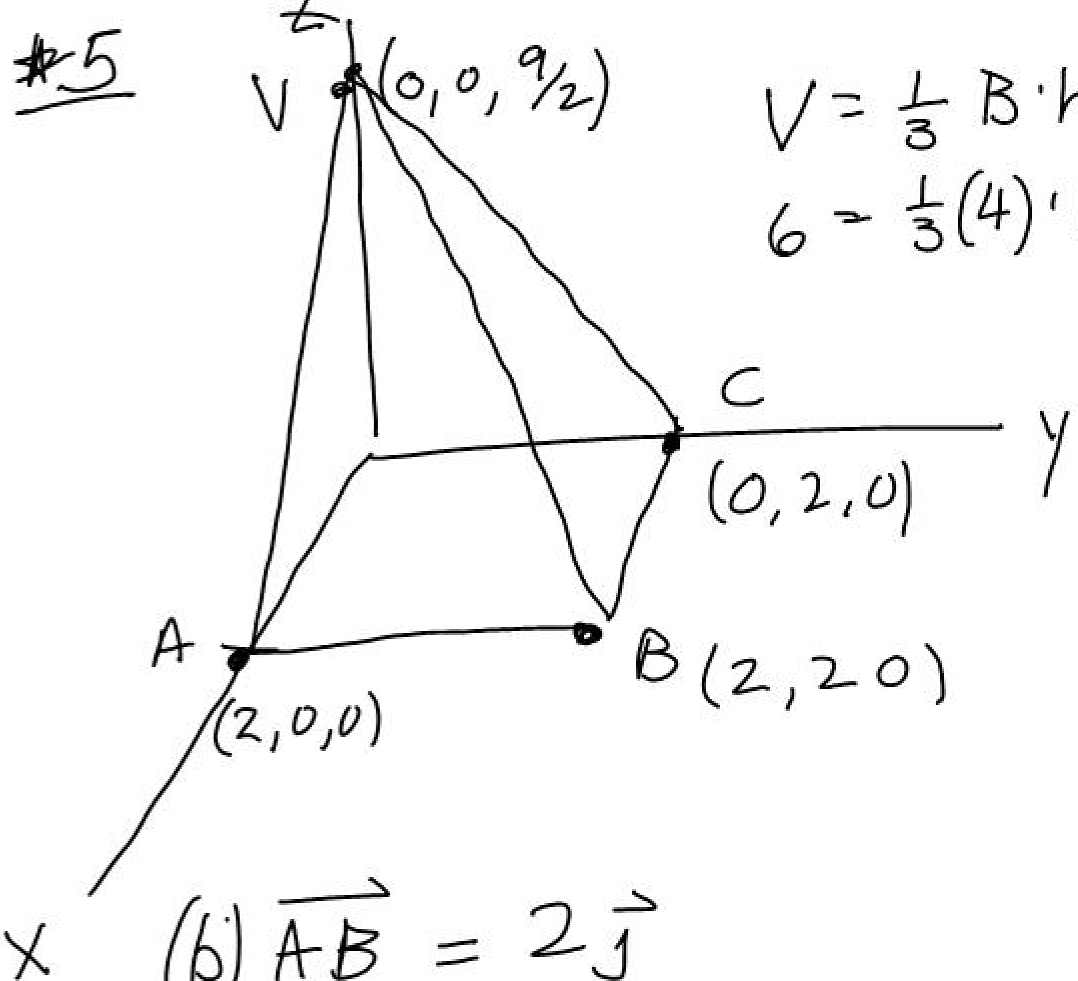
$z = 0$

$$\boxed{y = \boxed{0}}$$

plane yz

$x = 0$

#5



$$V = \frac{1}{3} B \cdot h$$

$$6 = \frac{1}{3}(4) \cdot h \rightarrow h = \frac{9}{2}$$

(b) $\overrightarrow{AB} = 2\vec{j}$

$$\overrightarrow{AV} = -2\vec{i} + \frac{9}{2}\vec{k}$$

$$\boxed{\begin{aligned} \vec{r} &= \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} -4 \\ 0 \\ 9 \end{pmatrix} \\ \vec{r} &= (2 - 4\beta)\vec{i} + (2\alpha)\vec{j} + (9\beta)\vec{k} \end{aligned}}$$

(c) plane BCV

$$\overrightarrow{BC} = \begin{pmatrix} -2 \\ 0 \\ 0 \end{pmatrix}$$

$$\overrightarrow{BV} = \begin{pmatrix} -2 \\ -2 \\ 9/2 \end{pmatrix}$$

$$8\vec{k} - (-18\vec{j})$$

$$\vec{n} = \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 0 & 0 \\ -4 & -4 & 9 \end{pmatrix} \begin{matrix} \vec{i} & \vec{j} \\ -2 & 0 \\ -4 & -4 \end{matrix} = 18\vec{j} + 8\vec{k} = \begin{pmatrix} 0 \\ 18 \\ 8 \end{pmatrix}$$

$$18y + 8z = \boxed{36}$$

$$18(2) + 8(0)$$

$$\boxed{9y + 4z = 18}$$

(d) $\overrightarrow{BV}$ dir. vector $= \begin{pmatrix} -2 \\ -2 \\ 9/2 \end{pmatrix}$

$$\boxed{\vec{r} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ -2 \\ 9/2 \end{pmatrix}}$$

(e) dir. vector: $\begin{pmatrix} -2 \\ 0 \\ 9/2 \end{pmatrix}$

$$\vec{r} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 0 \\ 9/2 \end{pmatrix}$$

$$x = 2 - 2\lambda \quad , \quad y = 0 \quad , \quad z = \frac{9}{2}\lambda$$

$$\lambda = \frac{x-2}{-2} \quad , \quad \lambda = \frac{z}{9/2}$$

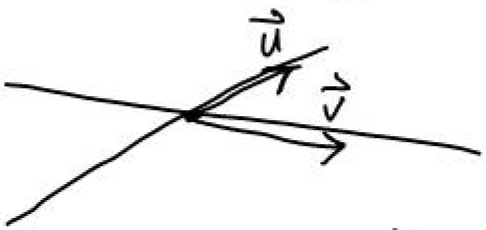
$$\boxed{\frac{x-2}{-2} = \frac{z-0}{9/2} \ , \ y = 0}$$

Lots of Formulas

• Angle between 2 lines

EX. $\vec{r}_1 = (2-\alpha)\vec{i} + (-1+2\alpha)\vec{j} + (1-\alpha)\vec{k}$

$$\vec{r}_2 = (2+\beta)\vec{j} + (3+\beta)\vec{k}$$



Find the angle between direction vectors.

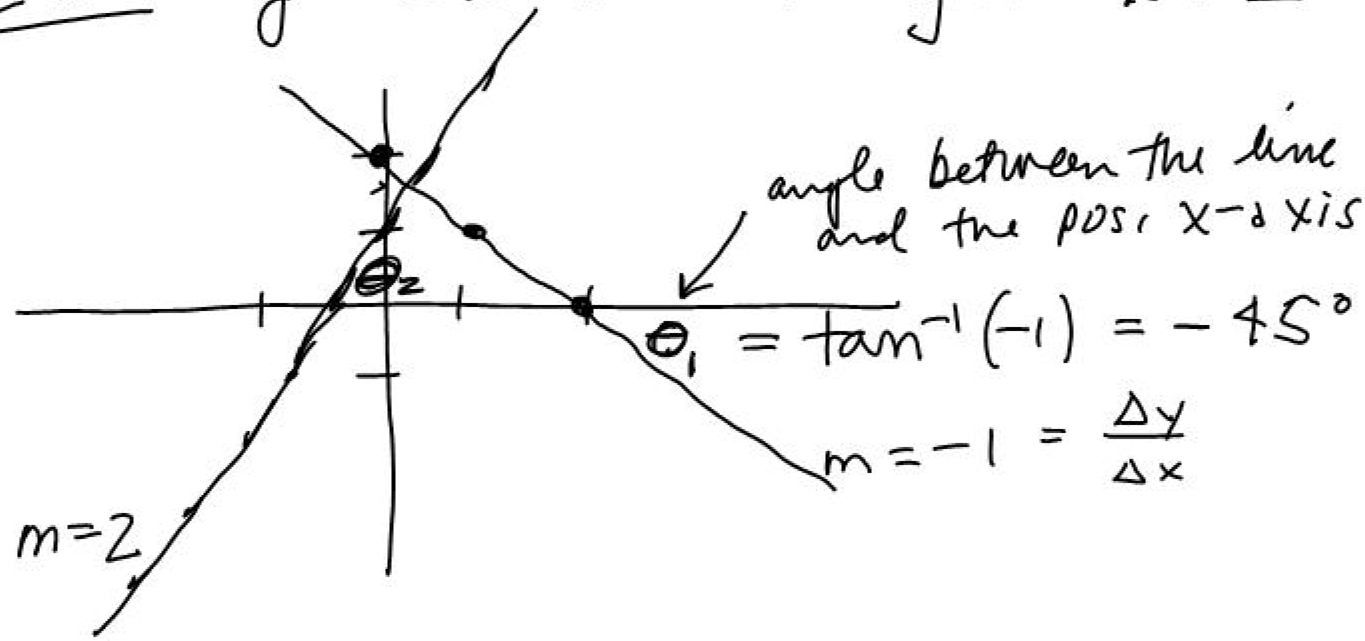
$$\vec{u} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} \qquad \cos\theta = \frac{\begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}}{\sqrt{6}\,\sqrt{2}}$$

$$\vec{v} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\cos\theta = \frac{1}{\sqrt{12}}$$

$$\theta = 73.2^\circ$$

Ex 2 $y = 2x + 1$ and $y = -x + 2$



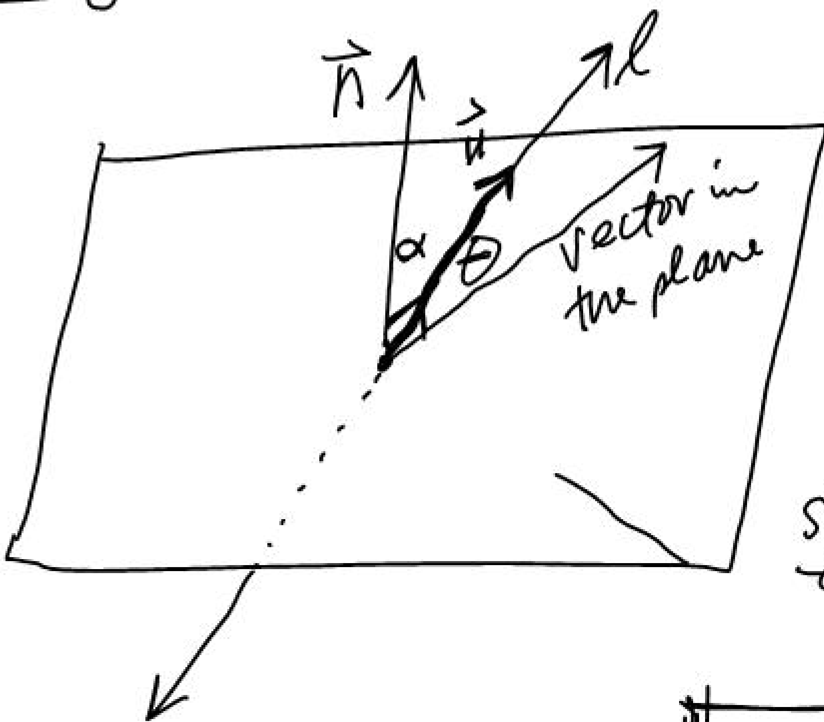
$\theta_2 = \arctan(2) = 63.4°$

The angle between two lines is

$$\boxed{\left|\arctan m_1 - \arctan m_2\right|}$$

$$= \left|-45° - 63.4°\right| = 108.4°$$

• Angle between a line and a plane



Angle between $\vec{n}$ and the dir. vector of the line:

$$\frac{\sin\theta}{\cancel{\cos\alpha}} = \frac{|\vec{n}\cdot\vec{u}|}{|\vec{n}|\,|\vec{u}|}$$

$$\sin x = \cos(90 - x)$$

$$\boxed{\theta = \sin^{-1}\left(\frac{|\vec{n}\cdot\vec{u}|}{|\vec{n}|\,|\vec{u}|}\right)}$$

Ex line: $\vec{r} = (2-\lambda)\vec{i} + \lambda\vec{j} + (4+\lambda)\vec{k}$

plane: $x - 3y + z = 2$

$\vec{u} = -\vec{i} + \vec{j} + \vec{k}$

$\vec{n} = \vec{i} - 3\vec{j} + \vec{k}$

$$\sin^{-1}\left(\frac{|-1-3+1|}{\sqrt{3}\,\sqrt{11}}\right)$$

$$= 31.5^\circ$$

- <u>Angle between 2 planes</u> = angle between the normal vectors

plane 1 : $2x - y - 3z = 4$

plane 2 : $\vec{r} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \alpha \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$

$\vec{n}_1 = \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$

$(2\vec{i} - 3\vec{j} + 3\vec{k}) - (-\vec{k} + 3\vec{i} + 6\vec{j})$

$\vec{n}_2 = \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & 3 \\ -1 & 1 & 2 \end{pmatrix} \begin{matrix} \vec{i} & \vec{j} \\ 3 & 1 \\ -1 & 1 \end{matrix} = -\vec{i} - 9\vec{j} + 4\vec{k}$

$\cos\theta = \left(\dfrac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| \, |\vec{n}_2|} \right)$

$$\boxed{\begin{array}{ll} \text{HW} & \text{11N \# 2} \\ & \text{11O \# 1–3} \\ & \text{11P \# 1, 2} \end{array}}$$