Planes

The Vector Product ("cross product")

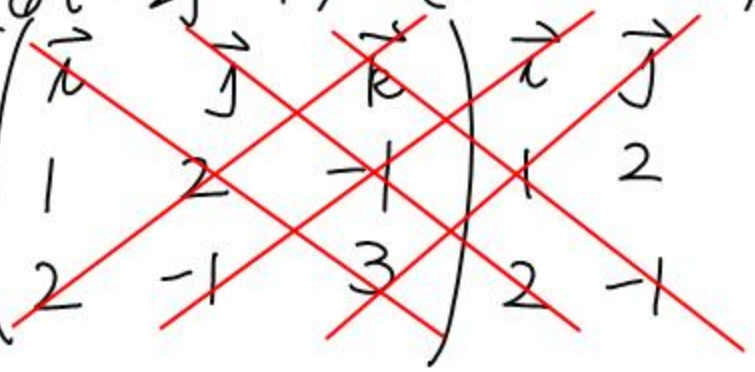
(3D vectors only)

$$\vec{u} = \vec{i} + 2\vec{j} - \vec{k}$$

$$\vec{v} = 2\vec{i} - \vec{j} + 3\vec{k}$$

$$\vec{u} \times \vec{v} = \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

$$(6\vec{i} - 2\vec{j} - \vec{k}) - (4\vec{k} + \vec{i} + 3\vec{j})$$



$$= 5\vec{i} - 5\vec{j} - 5\vec{k}$$

$$\vec{u} \cdot (\vec{u} \times \vec{v}) = (\vec{i} + 2\vec{j} - \vec{k}) \cdot (5\vec{i} - 5\vec{j} - 5\vec{k})$$

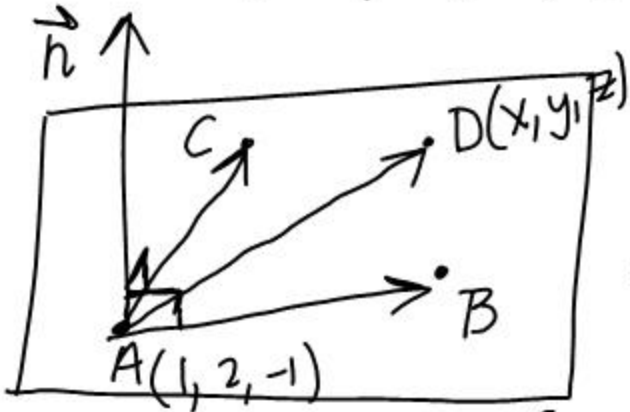
$$= 5 - 10 + 5 = 0$$

$$\vec{v} \cdot (\vec{u} \times \vec{v}) = (2\vec{i} - \vec{j} + 3\vec{b}) \cdot (5\vec{i} - 5\vec{j} - 5\vec{k})$$

$$= 10 + 5 - 15 = 0$$

$\vec{u} \times \vec{v}$ is <u>normal</u> to both $\vec{u}$ and $\vec{v}$

Ex. Write a Cartesian Equation for the plane through $A(1, 2, -1)$, $B(2, 3, 4)$, and $C(-1, 3, 5)$. "terminal − initial"



$\overrightarrow{AB} = \vec{i} + \vec{j} + 5\vec{k}$

$\overrightarrow{AC} = -2\vec{i} + \vec{j} + 6\vec{k}$

$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 5 \\ -2 & 1 & 6 \end{pmatrix} \begin{matrix} \vec{i} & \vec{j} \\ 1 & 1 \\ -2 & 1 \end{matrix}$

$(6\vec{i} - 10\vec{j} + \vec{k}) - (-2\vec{k} + 5\vec{i} + 6\vec{j})$

normal vector: $\vec{n} = \vec{i} - 16\vec{j} + 3\vec{k}$

$\overrightarrow{AD} = (x-1)\vec{i} + (y-2)\vec{j} + (z+1)\vec{k}$

$\vec{n} \cdot \overrightarrow{AD} = 0$

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$(\vec{i} - 16\vec{j} + 3\vec{k}) \cdot ((x-1)\vec{i} + (y-2)\vec{j} + (z+1)\vec{k}) = 0$

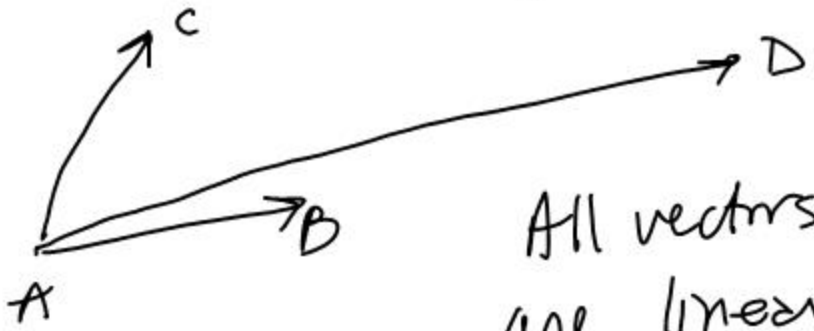
$x - 1 + (-16)(y-2) + 3(z+1) = 0$

$$\boxed{x - 16y + 3z = -34}$$

$1x - 16y + 3z$

Cartesian Eq. for the plane through A, B, and C.

Find the vector equation for this plane.



All vectors in this plane are linear combinations of $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

Ex. $3\overrightarrow{AB} - 2\overrightarrow{AC}$

$$\boxed{\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} + \beta \begin{pmatrix} -2 \\ 1 \\ 6 \end{pmatrix}}$$

point A $\quad \overrightarrow{AB} \quad \overrightarrow{AC}$

$$\vec{r} = (1+\alpha-2\beta)\vec{i} + (2+\alpha+\beta)\vec{j} + (-1+5\alpha+6\beta)\vec{k}$$

parametric eqs

$$\begin{cases} x = 1+\alpha-2\beta \\ y = 2+\alpha+\beta \\ z = -1+5\alpha+6\beta \end{cases}$$

Ex. Find a Cartesian Equation for the plane through $P(2,4,1)$, $Q(1,1,1)$ and $R(3,2,2)$

$(-3\vec{i} + 2\vec{k}) - (-3\vec{k} - \vec{j})$

$$\vec{n} = \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -3 & 0 \\ 1 & -2 & 1 \end{pmatrix} \begin{matrix} \vec{i} & \vec{j} \\ -1 & -3 \\ 1 & -2 \end{matrix} = -3\vec{i} + \vec{j} + 5\vec{k}$$

$$-3x + y + 5z = 3$$

plug in Q $\quad -3(1) + (1) + 5(1)$

plug in P $\quad -3(2) + (4) + 5(1) = 3$

HW $\quad U(-2, 4, 6)$

$V(1, 2, 1)$

$W(3, 3, 4)$

Write the vector, parametric, and Cartesian equations for the plane through U, V, and W.