points : A(1, 3) and B(4, 9)

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} = 3\vec{i} + 6\vec{j}$ ← the direction vector for the line

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \vec{i}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \vec{j}$

vector equation

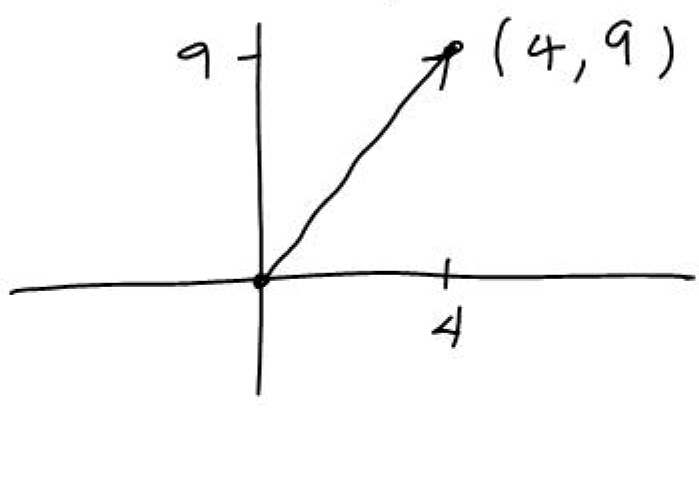
$$\boxed{\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 6 \end{pmatrix}}$$

↑ parameter

$t = 1$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + 1\begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$$

↙ position vector for (4, 9)



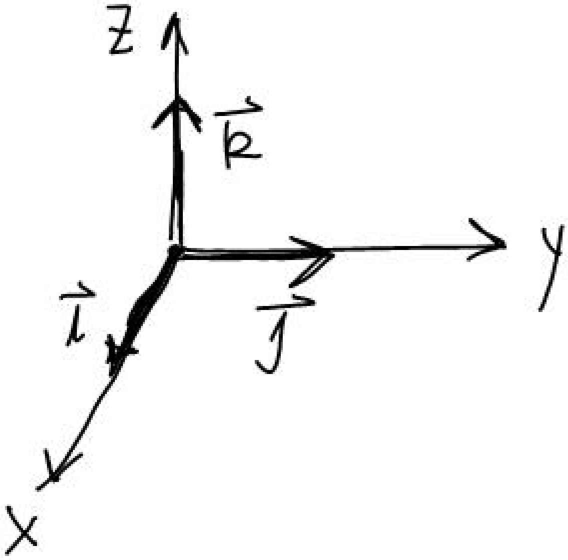
$t = -2$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + (-2)\begin{pmatrix} 3 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} -5 \\ -9 \end{pmatrix}$$

Ex. Write a vector eq. for the line through $(-1, 2, 5)$ and $(3, 4, 9)$

direction vector: $\begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix} = 4\vec{i} + 2\vec{j} + 4\vec{k}$



$$\left.\begin{aligned} \vec{i} &= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \\ \vec{j} &= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \\ \vec{k} &= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \end{aligned}\right\} \text{standard unit vectors}$$

linear equation

$$\boxed{\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix}}$$

Vector equation

parametric Eqs

$$\begin{cases} x = -1 + 4\lambda \\ y = 2 + 2\lambda \\ z = 5 + 4\lambda \end{cases}$$

x, y, and z

Cartesian form of the Equation

$x = -1 + 4\lambda$	$y = 2 + 2\lambda$	$z = 5 + 4\lambda$
$x + 1 = 4\lambda$	$y - 2 = 2\lambda$	$z - 5 = 4\lambda$
$\frac{x+1}{4} = \lambda$	$\frac{y-2}{2} = \lambda$	$\frac{z-5}{4} = \lambda$

Cartesian Eq: $\boxed{\frac{x+1}{4} = \frac{y-2}{2} = \frac{z-5}{4}}$

Write the Cartesian Eq. for the line through $(2, 4, -5)$ and $(1, 3, 8)$

Direction vector $\begin{pmatrix} -1 \\ -1 \\ 13 \end{pmatrix}$

$$\boxed{\frac{x-2}{-1} = \frac{y-4}{-1} = \frac{z+5}{13}}$$

Ex. $P(-2, 5)$ and $Q(1, 4)$

Write the vector, parametric, and Cartesian Equations for the line.

direction vector: $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

slope

$$m = \frac{4-5}{1--2} = \frac{-1}{3}$$

$$y - 5 = -\frac{1}{3}(x+2)$$

$$y = -\frac{1}{3}x + \frac{13}{3}$$

vector eq.

$$\boxed{\vec{r} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \end{pmatrix}}$$

parametric eqs.

$$\boxed{\begin{cases} x = -2 + 3\lambda \\ y = 5 - \lambda \end{cases}}$$

Cartesian eqs

$$\lambda = \frac{x+2}{3}$$

$$\lambda = \frac{y-5}{-1}$$

$$\boxed{\frac{x+2}{3} = \frac{y-5}{-1}}$$

$$-x - 2 = 3y - 15$$

$$3y = -x + 13$$

$$y = -\frac{1}{3}x + \frac{13}{3}$$

The norm or length or magnitude of a vector.

$$\vec{u} = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}$$

$$|\vec{u}| = \sqrt{2^2 + 1^2 + (-4)^2} = \sqrt{21}$$

The "dot" product or the scalar product.

$$\vec{u} = 3\vec{i} - 4\vec{j} \qquad \vec{v} = -\vec{i} + 2\vec{j}$$

$$\vec{u} \cdot \vec{v} = (3)(-1) + (-4)(2) = -11$$

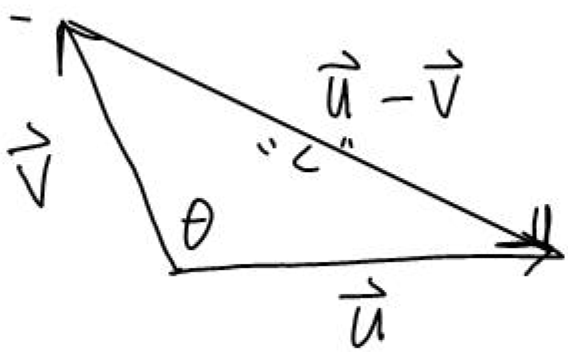
$$\vec{w} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \qquad \vec{z} = \begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$$

$$\vec{w} \cdot \vec{z} = (-2)(1) + (1)(4) + (3)(-5) = -13$$

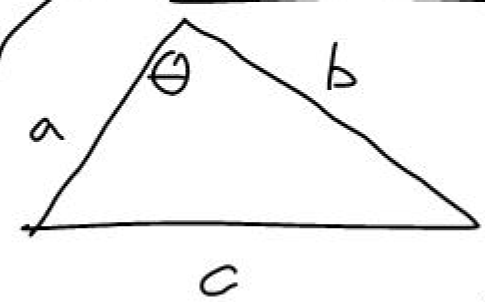
Ex. $\vec{u} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$

$$\vec{u}\cdot\vec{u} = 1^2 + (-2)^2 + 3^2 = 14$$

$$\boxed{\vec{u}\cdot\vec{u} = |\vec{u}|^2} \ *$$



Cosine Rule



$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$\cos\theta = \frac{a^2+b^2-c^2}{2ab}$$

$$\cos\theta = \frac{|\vec{u}|^2 + |\vec{v}|^2 - |\vec{u}-\vec{v}|^2}{2|\vec{u}||\vec{v}|}$$

$$\begin{aligned} |\vec{u}-\vec{v}|^2 &= (\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v}) \\ &= \vec{u}\cdot\vec{u} - \vec{u}\cdot\vec{v} - \vec{v}\cdot\vec{u} + \vec{v}\cdot\vec{v} \\ &= |\vec{u}|^2 - 2\vec{u}\cdot\vec{v} + |\vec{v}|^2 \end{aligned}$$

$$\cos\theta = \frac{\cancel{|\vec{u}|^2} + \cancel{|\vec{v}|^2} - \left(\cancel{|\vec{u}|^2} - 2\vec{u}\cdot\vec{v} + \cancel{|\vec{v}|^2}\right)}{2|\vec{u}|\,|\vec{v}|}$$

$$\boxed{\cos\theta = \frac{\cancel{2}\,\vec{u}\cdot\vec{v}}{\cancel{2}\,|\vec{u}|\,|\vec{v}|}}$$

The Cosine Rule for angles between vectors

- If $\vec{u}\cdot\vec{v} > 0$, θ is acute
- If $\vec{u}\cdot\vec{v} < 0$, θ is obtuse

* If $\vec{u}\cdot\vec{v} = 0$, $\vec{u}$ and $\vec{v}$ are <u>normal</u> (perpendicular)

<u>Exercises</u>

① $A(2,4,-1)$ $B(-1,5,2)$

ⓐ $|\overrightarrow{AB}|$ ⓑ $\overrightarrow{AB}\cdot\overrightarrow{AB}$ ⓒ Write the equation for $\overleftrightarrow{AB}$ in vector, parametric and Cartesian forms.

(2) $\vec{u} = 3\vec{i} - 4\vec{j} + \vec{k}$

$\vec{v} = \vec{i} + \vec{j} + 2\vec{k}$

$\vec{w} = -\vec{i} - 2\vec{j} - 3\vec{k}$

(a) $\vec{u} \cdot \vec{v}$ (b) $\vec{v} \cdot \vec{w}$ (c) $|\vec{u}|$ (d) $|\vec{w}|$

(e) Find the angle between $\vec{u}$ and $\vec{v}$ to the nearest tenth of a degree