#1 X = errors/page

$$X \sim P_o(3)$$

$$\boxed{P(X=x) = \frac{e^{-m} m^x}{x!}}$$

(a) $P(X=1) = \dfrac{e^{-3} 3^1}{1!} = \dfrac{3}{e^3}$ or $3e^{-3}$

(b) Y = errors on 10pp

$$Y \sim P_o(30)$$

$$P(Y=25) = \frac{e^{-30} \cdot 30^{25}}{25!}$$

$\boxed{\#2}$ X = number of ⚀ $\qquad X \sim B(3, 1/4)$

$$P(X=0) = \binom{3}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^3 = \frac{27}{64}$$

$$P(X=1) = \binom{3}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^2 = \frac{27}{64}$$

$$P(X=2) = \binom{3}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^1 = \frac{9}{64}$$

$$P(X=3) = \binom{3}{3}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^0 = \frac{1}{64}$$

(3) 2% faulty widgets X = # of faulty widgets

$X \sim B(25, 0.02)$

(a) $P(X=0)$ = binomialpdf$(25, 0.02, 0)$

$= 0.603$

(b) $P(X=1)$ = binomialpdf$(25, 0.02, 1)$

$= 0.308$

(c) $P(X \geq 1) = 1 - P(X=0)$

$= 1 - 0.603 = 0.397$

(d) $E(X) = np = 25(0.02) = 0.5$

$Var(X) = npq = 25(0.02)(0.98) = 0.49$

$\sigma = 0.7$

(#4) $X \sim P_o(0.2)$

(a) Y = flaws / 100 yd^2

$Y \sim P_o(20)$

$P(Y > 12) = 1 - P(Y \leq 12)$

$= 1 - \text{poissoncdf}(20, 12)$

$= 0.961$

(b) $P(Y < 20) = P(Y \leq 19)$

$= \text{poissoncdf}(20, 19)$

$= 0.470$

(c) X = flaws / yd^2

$Var(X) = m = 0.2$

$\sigma = \sqrt{0.2} = 0.447$

$\boxed{10\ \mathrm{I}}$ #3. $f(x) = 6x(1-x), \quad 0 < x < 1$

(a) $E(X) = \int_0^1 x \cdot 6x(1-x)\,dx$

$$= \int_0^1 (6x^2 - 6x^3)\,dx$$

$$= \left[2x^3 - \frac{3}{2}x^4\right]_0^1$$

$$= \left(2 - \frac{3}{2}\right) - (0) = \frac{1}{2}$$

(b) $Var(X) = \int_0^1 (x-\mu)^2 \cdot f(x)\,dx$

$$= \int_0^1 \left(x - \frac{1}{2}\right)^2 \cdot (6x(1-x))\,dx$$

$$= 0.05$$

(b*) discrete distribution:

$$\mathrm{Var}(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \int_0^1 x^2 \cdot 6x(1-x)\,dx$$

$$= \int_0^1 (6x^3 - 6x^4)\,dx$$

$$= \left[\frac{3}{2}x^4 - \frac{6}{5}x^5\right]_0^1 = \frac{3}{2} - \frac{6}{5} = \frac{3}{10}$$

$$\mathrm{Var}(X) = E(X^2) - [E(X)]^2$$

$$= \frac{3}{10} - \left(\frac{1}{2}\right)^2 = \frac{6}{20} - \frac{5}{20}$$

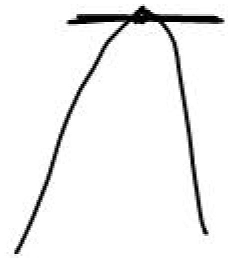
$$= \frac{1}{20}$$

(c) $\int_0^M 6x(1-x)\,dx = \frac{1}{2}$

$M = 0{,}5$

(d) $f(x) = 6x(1-x) = 6x - 6x^2$

$f'(x) = 6 - 12x = 0$



Mode: $x = \frac{1}{2}$

$P(X = \frac{1}{2}) \approx 0$

$P(0.49 < X < 0.51) = \int_{0{,}49}^{0{,}51} f(x)\,dx$

Cumulative Probability Functions

(cdf)

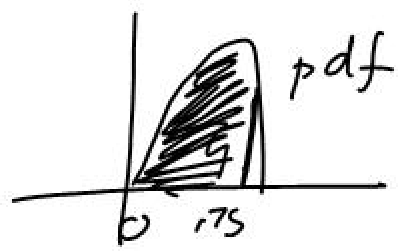
If the pdf for X is f(x) on [0, 2]

then the cdf for X is $F(x) = \int_0^x f(t)\,dt$

Ex. $\overset{\text{pdf}}{f(x)} = 6x(1-x)$ on [0, 1]

cdf $F(x) = \int_0^x \overset{6t-6t^2}{6t(1-t)}\,dt = \left[3t^2 - 2t^3\right]_0^x$

$$\boxed{F(x) = 3x^2 - 2x^3}$$



$P(X \le 0.75) = F(0.75) =$

$P(X \ge 0.75) = 1 - F(0.75) =$

HW The pdf for X is

$$f(x) = \frac{1}{2} - \frac{x^2}{b} \text{ on } [0, 3].$$

a) Find b

b) Find $P(0 < X < 0.5)$

c) Find the mean

d) Find the variance

e) Find the median (calculator)

f) Find the mode

g) Find $F(x)$, the cdf for X