$\boxed{10D}$ #3. $X \sim B(8, 0.01)$

a) i) $P(X \geq 1) = 1 - P(X \leq 0) = 0.0773$

ii) $P(X \leq 2) = \text{binomialcdf}(8, 0.01, 2)$

$= 0.9999$

b) $P(\text{2 defects} \mid \text{at least 1 defect})$

$$= \frac{P(\text{2 defects} \cap \text{at least 1 defect})}{P(\text{at least 1 defect})}$$

$$= \frac{P(X=2)}{P(X \geq 1)} = \frac{0.002636}{0.077255}$$

$= 0.0341$

#5. $X \sim B(n, 0.4)$

$n=2$ $B(2, 0.4)$

X	0	1	2
$P(X \le x)$	0.36	0.84	1

$n=5$

X	0	1	2	3	4	5
$P(X \le x)$						

$X \sim B(10, 1/2)$

#(a) $P(X = 4)$ = binomialpdf(10, 1/2, 4)

b) $P(X \ge 6) = 1 - P(X \le 5)$

$= 1 -$ binomialcdf (10, 1/2, 5)

c) $P(X \le 5)$ = binomialcdf (10, 1/2, 5)

The mean of a binomial distribution:

$\mu = E(X) = np$

expected value

$X \sim B(10, 1/2)$

$E(X) = 10\left(\frac{1}{2}\right) = 5$

$\sigma^2 = Var(X) = npq$

$Var(X) = 10\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{5}{2}$

std. dev.

$\sigma = \sqrt{\frac{5}{2}} \approx 1.58$

Median

The average of the largest outcome with <50% cumulative prob. and the smallest outcome with >50% cumulative prob.

$X \sim B(3, 1/6)$

X	0	1	2	3
$P(X \le x)$	0.5787	0.926	0.995	1

median = 0

$X \sim B(5, 1/6)$

cdf

X	0	1	2	3	4	5
$P(X \le x)$	0.40	0.80	0.96	0.996	0.999	1

median = 1/2

Mode

$X \sim B(5, 1/6)$

X	0	1	2	3	4	5
$P(X=x)$	0.402	0.402	0.161	0.03	0.003	0.0001

pdf

Bimodal: The modes are 0 and 1

$\boxed{10E}$ #6 (a) X = number correct out of 20

$X \sim B(20, 1/4)$

a) i) $P(X=0) = 0.00317$

ii) $P(X>10) = 1 - P(X \le 10) = 0.00394$

iii) $P(X \le 5) =$ binomial cdf$(20, 0.25, 5)$

$= 0.617$

b) $E(X) = (20)\left(\frac{1}{4}\right) = 5$

$Var(X) = (20)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right) = \frac{15}{4} = 3.75$

$\sigma = \frac{\sqrt{15}}{2} \approx 1.94$

c) Y = number of students getting more than 10 (out of 5 students)

$Y \sim B(5,\ 0.00394)$

$P(Y \geq 2) = 1 - P(Y \leq 1)$

$= 0.000154$

The Poisson Distribution

- Events are independent
- Events occur in small time interval or a small space with a fixed probability
- Events cannot occur simultaneously

$X \sim P_o(m)$.

$$\boxed{P(X=x) = \frac{e^{-m} m^x}{x!}}$$

This 1 parameter is the <u>mean</u> outcome.

Ex. The errors per page in a manuscript have a Poisson distribution. There are 2 errors per page on average. $\boxed{P_o(2)}$

a) $P(X=2) = \dfrac{e^{-2} \cdot 2^2}{2!} = \dfrac{2}{e^2} = 0.271$

b) $P(X=5) = \dfrac{e^{-2} \cdot 2^5}{5!} = \dfrac{32}{120 e^2}$

$= \dfrac{4}{15 e^2} = 0.0361$

c) $P(X<4) = P(X \leq 3)$

$= \text{poissoncdf}(2, 3)$

$= 0.857$

Ex. A lawn has on average 4 dandelions per square yard. X = number of dandelions

a) $P(X=0)$ = poissonpdf(4, 0) = 0.0183

b) $P(X>5) = 1 - P(X \leq 5)$

= 1 − poissoncdf(4, 5)

= 0.215

HW $\boxed{10E}$ # 1, 2, 8

$\boxed{10F}$ # 2

$\boxed{10H}$ # 2
