Warm up :

① $\int \frac{x}{\sqrt{x^2+1}}\,dx$

② $\int \frac{1}{\sqrt{x^2+1}}\,dx$

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① $\frac{1}{2}\int \frac{2x}{\sqrt{x^2+1}}\,dx = \frac{1}{2}\int u^{-\frac{1}{2}}\,du$

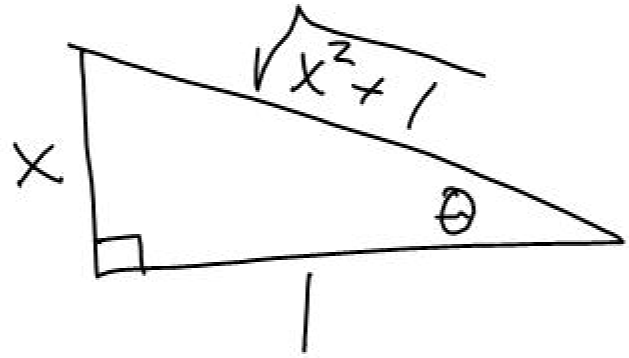
$u = x^2+1$

$du = 2x\,dx$

$= u^{1/2} + C$

$= \underline{\underline{\sqrt{x^2+1} + C}}$

$$\int \frac{1}{\sqrt{x^2+1}}\,dx$$




$$\frac{x}{1} = \tan\theta$$

$$dx = \sec^2\theta\, d\theta$$

$$= \int \frac{\sec^2\theta\, d\theta}{\sqrt{\underset{\sec^2\theta}{\cancel{\tan^2\theta+1}}}}$$

$$= \int \sec\theta\, d\theta$$

$$= \ln\left|\sec\theta + \tan\theta\right| + C$$

$$= \underline{\ln\left|\sqrt{x^2+1} + x\right| + C}$$

## Area

$\boxed{\#1}$ $\displaystyle\int_0^{\pi} x^2 \sin x \, dx$

| $u$ | $dv$ |
|---|---|
| $x^2$ | $\sin x$ |
| $(-1)\,2x$ | $-\cos x$ |
| $2$ | $-\sin x$ |
| $(-1)\,0$ | $\cos x$ |

$$= \left[ -x^2 \cos x + \cancel{2x \sin x}^{\,0} + 2\cos x \right]_0^{\pi}$$

$$= \left(-\pi^2(-1) + 2(-1)\right) - \left(2(1)\right)$$

$$= \pi^2 - 4$$

1. u-subst.
2. parts
   - $x^n \sin x$
   - $x^n \cos x$
   - $x^n e^x$
   - $x^n \ln x$
3. partial fracs
   - fractions with denom. that factor
4. trig subst.
   - $x^2 + a^2$
   - $x^2 - a^2$
   - $a^2 - x^2$

#5. $\int_0^1 \left( 2 - x - \frac{1}{x^2+3x+2} \right) dx$

$= \left[ 2x - \frac{1}{2}x^2 + \ln|x+2| - \ln|x+1| \right]_0^1$

$= \left( \frac{3}{2} + \ln 3 - \ln 2 \right) - \left( \ln 2 \right)$

$= \underline{\frac{3}{2} + \ln 3 - 2\ln 2}$

$\frac{1}{x^2+3x+2} = \frac{A}{x+2} + \frac{B}{x+1}$

$1 = A(x+1) + B(x+2)$

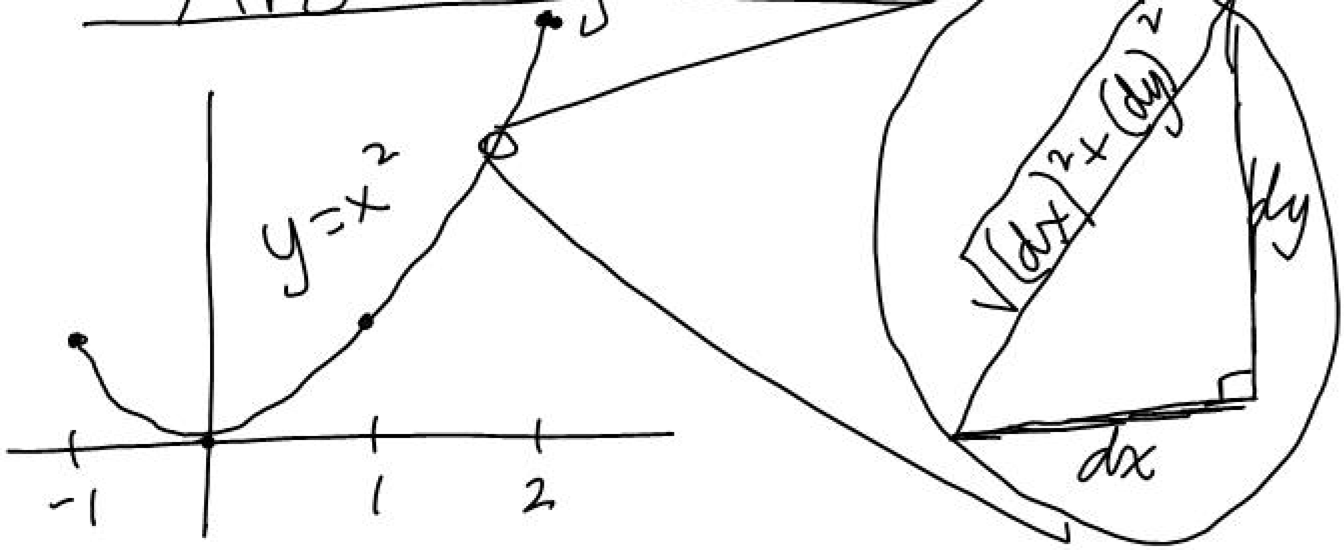
$x=-2$ $\quad 1 = A(-1) \rightarrow A = -1$

$x=-1$ $\quad 1 = B(1) \rightarrow B = 1$

$\int \frac{-1}{x+2}\,dx + \int \frac{1}{x+1}\,dx$

$= -\ln|x+2| + \ln|x+1| + C$

# Arc Length




$$\int_{-1}^{2} \sqrt{(dx)^2+(dy)^2} = \int_{-1}^{2} \sqrt{\frac{(dx)^2+(dy)^2}{(dx)^2}}\, dx$$

$$= \int_{-1}^{2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$y = x^2$

$\frac{dy}{dx} = 2x$

$$= \int_{-1}^{2} \sqrt{1+4x^2}\, dx$$ ← use trig subst.

$\approx$

Arclength $\displaystyle\int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

or $\displaystyle\int_{y_1}^{y_2} \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$

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HW find the length of $y = \ln(\sec x)$ on $\left[-\pi/4, \pi/4\right]$.

Area Between Curves #4, #8