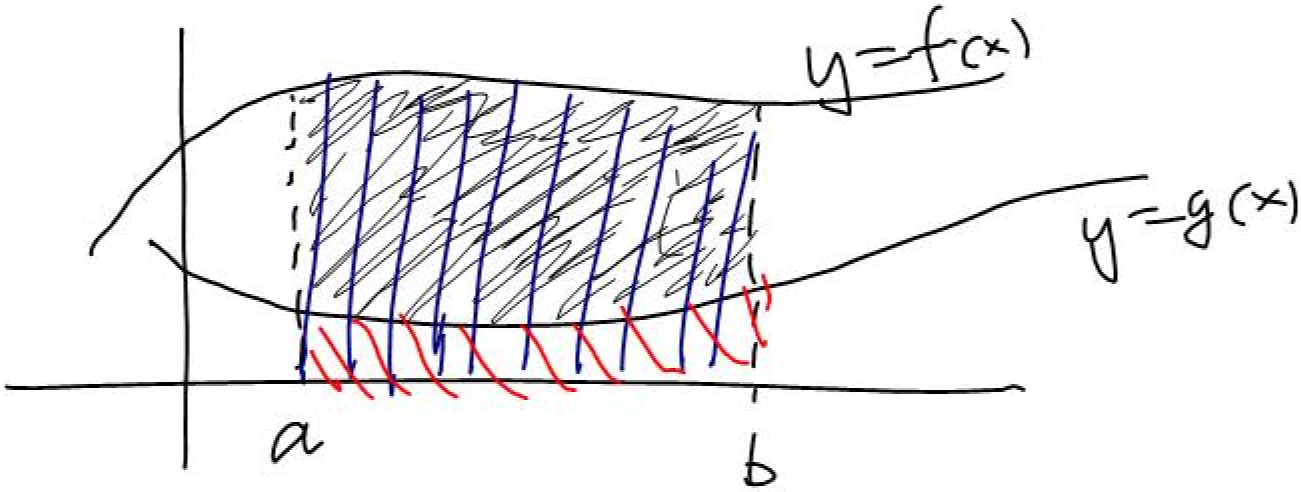
Area between Curves

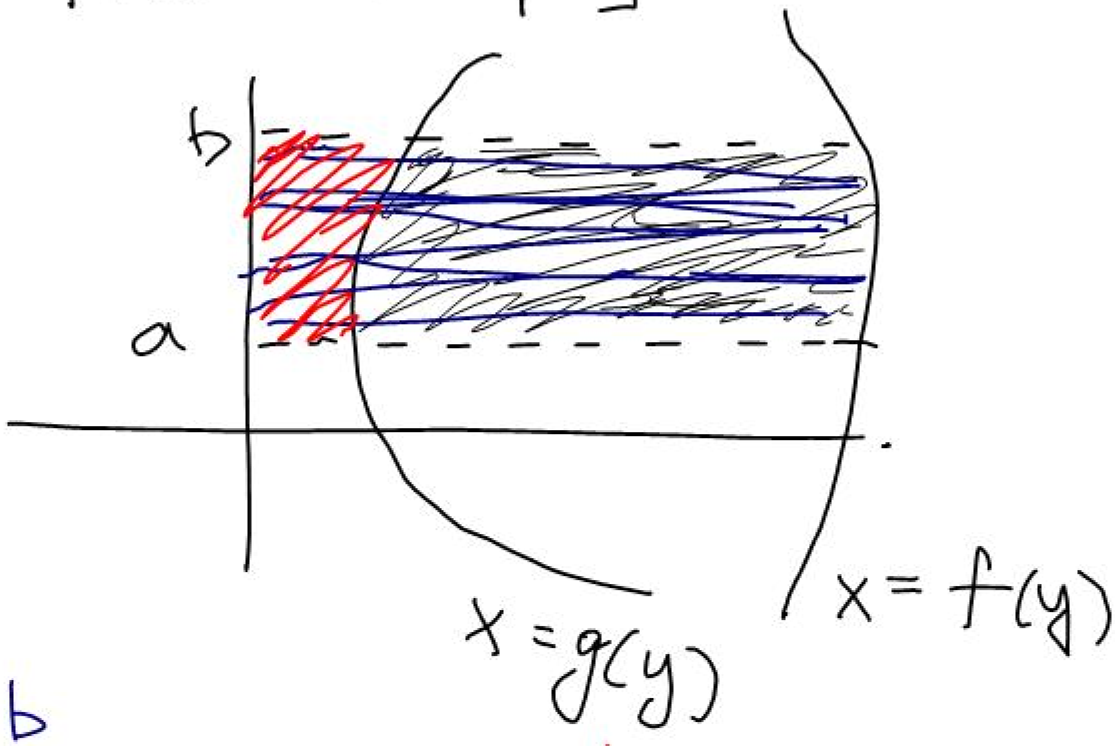


$$\underline{\text{area}} = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

$$= \int_a^b [f(x) - g(x)]\,dx$$

top curve − bottom curve

Functions of y



$$\int_a^b f(y)\,dy - \int_a^b g(y)\,dy$$

$$= \int_a^b \left[f(y) - g(y) \right] dy$$

right curve − left curve

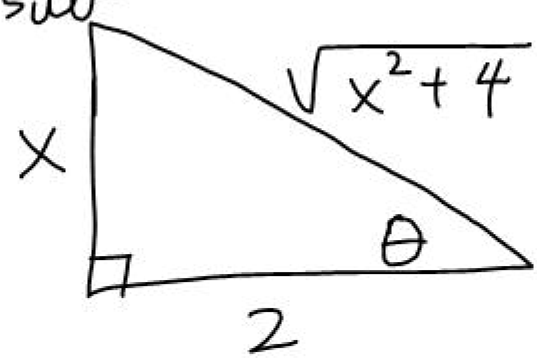
$\boxed{\#3}$ Which curve is the "top" curve?

> plug in $x = 0$

$$\int_0^1 \left[\frac{1}{\sqrt{x^2+4}} - \frac{x}{\sqrt{x^2+4}} \right] dx$$

Find an antiderivative + use the Fundamental Theorem

$$\int \frac{dx}{\sqrt{x^2+4}}$$ ← use trig sub



$$= \int \frac{2\sec^2\theta \, d\theta}{\underbrace{\cancel{\sqrt{4\tan^2\theta+4}}}_{2\sec\theta}}$$

$\frac{x}{2} = \tan\theta$ ⇐

$x = 2\tan\theta$

$dx = 2\sec^2\theta$ –

$$= \int \sec\theta \, d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C_1$$

$$= \ln\left| \frac{\sqrt{x^2+4}}{2} + \frac{x}{2} \right| + C_1 = \ln \frac{|\sqrt{x^2+4} + x|}{2}$$

$$= \ln|\sqrt{x^2+4}+x| - \ln 2$$

$$= \ln|\sqrt{x^2+4} + x| + C$$

$$\frac{1}{2}\int \frac{2x}{\sqrt{x^2+4}}\,dx = \frac{1}{2}\int u^{-1/2}\,du$$

$u = x^2+4$

$du = 2x\,dx$

$$= u^{1/2} + C$$

$$= \sqrt{x^2+4} + C$$

$$\int_0^1 \left[\frac{1}{\sqrt{x^2+4}} - \frac{x}{\sqrt{x^2+4}}\right]dx$$

$$= \Big[\ln\left|\sqrt{x^2+4}+x\right| - \sqrt{x^2+4}\Big]_0^1$$

$$= \left(\ln\left|\sqrt{5}+1\right| - \sqrt{5}\right) - \left(\ln\left|\sqrt{4}+0\right| - 2\right)$$

$$= \ln\left(\frac{\sqrt{5}+1}{2}\right) - \sqrt{5} + 2$$

$\ln a - \ln b = \ln \frac{a}{b}$

#7

$$\int_{-1}^{2} \left[(y+1) - (y^2-1) \right] dy$$

$$= \int_{-1}^{2} \left[-y^2 + y + 2 \right] dy$$

$$= \left[-\frac{1}{3}y^3 + \frac{1}{2}y^2 + 2y \right]_{-1}^{2}$$

$$= \left(-\frac{8}{3} + 2 + 4 \right) - \left(\frac{1}{3} + \frac{1}{2} - 2 \right)$$

$$= -3 + 6 + 2 - \frac{1}{2} = 4.5$$

$$-\int \frac{-\sin\theta}{4+\cos^2\theta}\,d\theta = -\int \frac{du}{4+u^2}$$

$$u = \cos\theta$$
$$du = -\sin\theta\,d\theta$$

$$= -\frac{1}{2}\tan^{-1}\left(\frac{\cos\theta}{2}\right) + C$$