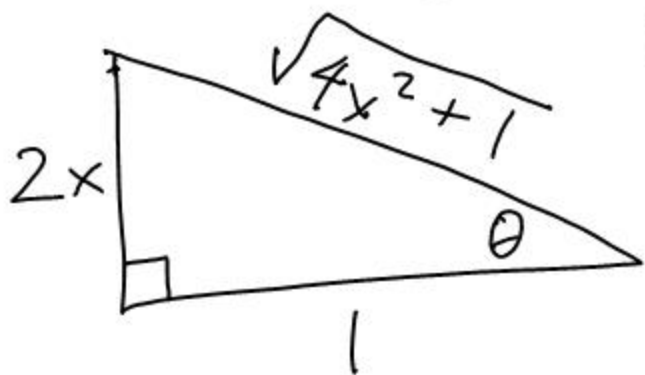


# More Trig Substitution

$$\begin{cases} x^2 + a^2 \\ x^2 - a^2 \\ a^2 - x^2 \end{cases}$$

$$\int \frac{dx}{\sqrt{4x^2 + 1}}$$
$$= \int \frac{\frac{1}{2} \sec^2 \theta d\theta}{\sqrt{\cancel{\tan^2 \theta + 1}^{\sec^2 \theta}}}$$



legs  $\begin{cases} 2x \\ 1 \end{cases}$

set up a trig ratio  $\rightarrow \frac{2x}{1} = \tan \theta \leftarrow$   
(don't use the side with  $\sqrt{\quad}$ )  $\rightarrow x = \frac{1}{2} \tan \theta$   
solve for  $x$   $\rightarrow dx = \frac{1}{2} \sec^2 \theta d\theta$   
compute  $dx$

$$\frac{1}{2} \int \frac{\sec^2 \theta}{\sec \theta} d\theta = \frac{1}{2} \int \sec \theta d\theta$$

$$= \frac{1}{2} \ln |\sec \theta + \tan \theta| + C$$

$$= \frac{1}{2} \ln |\sqrt{4x^2 + 1} + 2x| + C$$

$$\sqrt{9+16} = \sqrt{25} = 5$$

$$3 + 4 = 7$$

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On the calculator:

$$\sec^{-1} x \longrightarrow \cos^{-1}\left(\frac{1}{x}\right)$$

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$$\text{Ex } \int \frac{dx}{(x^2 + 1)^{3/2}}$$

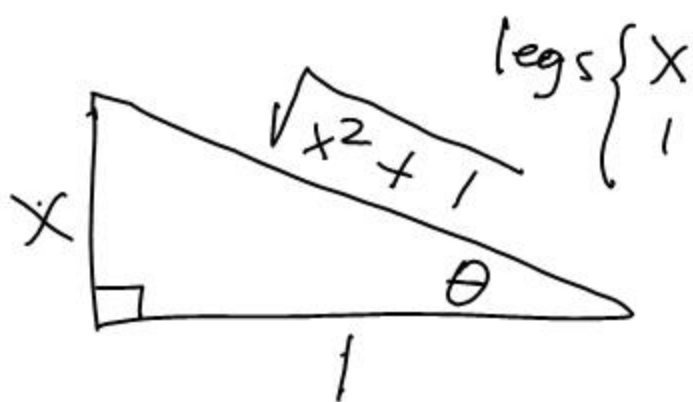
$$= \int \frac{\sec^2 \theta d\theta}{(\cancel{\tan^2 \theta} + 1)^{3/2}}$$

$\sec^2 \theta$

$$= \int \frac{\sec^2 \theta}{\sec^3 \theta} d\theta$$

$$= \int \cos \theta d\theta$$

$$= \sin \theta + C = \frac{x}{\sqrt{x^2 + 1}} + C$$



$$\frac{x}{1} = \tan \theta$$

$$dx = \sec^2 \theta d\theta$$

$$\frac{1}{\sec \theta} = \cos \theta$$

check  $\frac{d}{dx} \left[ \frac{x}{\sqrt{x^2 + 1}} \right] =$

$$= \frac{(x^2 + 1)^{1/2} - x \cdot \frac{1}{2} (x^2 + 1)^{-1/2} \cdot 2x}{x^2 + 1}$$

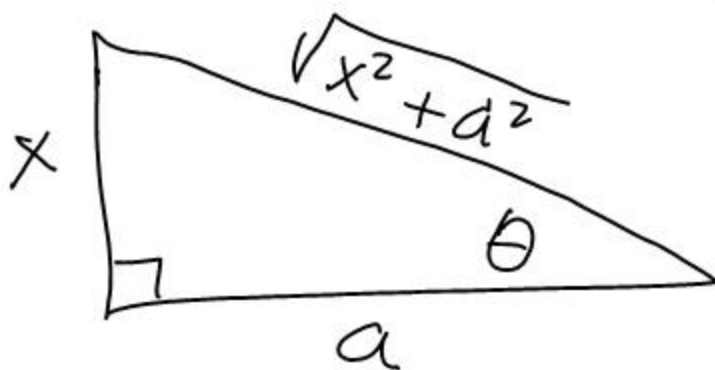
$$= \frac{(x^2+1)^{1/2} - x \cdot \frac{1}{2} (x^2+1)^{-1/2} \cdot 2x}{x^2+1}$$

$$= \frac{(x^2+1)^{-1/2} \left[ \overset{1}{(x^2+1)^1} - x^2 \right]}{(x^2+1)^1}$$

$$= \frac{1}{(x^2+1)^{1-1/2}} = \frac{1}{(x^2+1)^{3/2}}$$

The 3 possible triangles

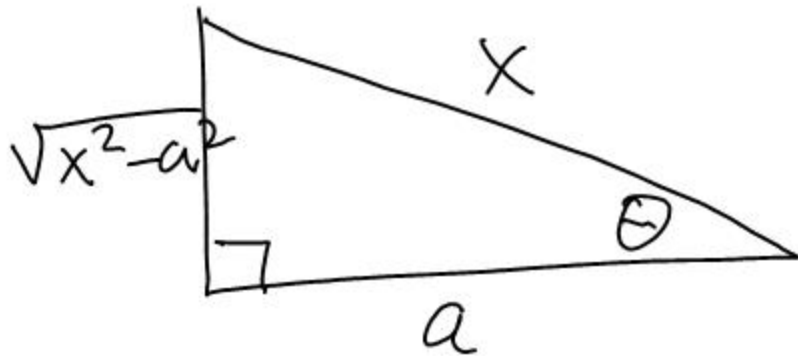
$$x^2 + a^2$$



$$\frac{x}{a} = \tan \theta$$

$$x^2 - a^2$$

↑                    ↑  
hyp<sup>2</sup> - leg<sup>2</sup>

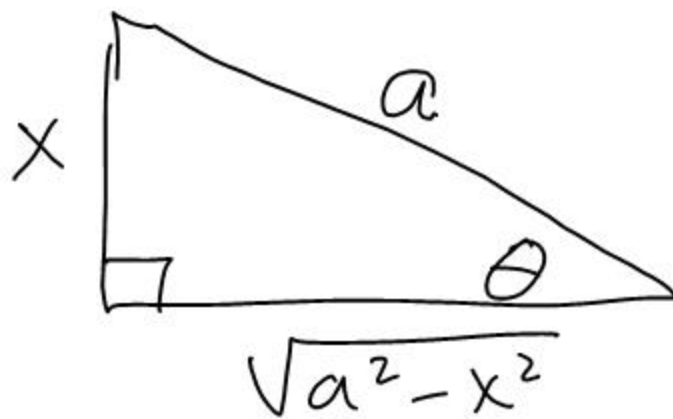


$$\frac{x}{a} = \sec \theta$$

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$$a^2 - x^2$$

↑                    ↑  
hyp<sup>2</sup> - leg<sup>2</sup>



$$\frac{x}{a} = \sin \theta$$