(d) $\frac{1}{2}\int \frac{2(x-3)}{x^2-6x+8}dx = \frac{1}{2}\int \frac{du}{u}$

$u = x^2-6x+8$ $\quad = \frac{1}{2}\ln|x^2-6x+8| + C$

$du = (2x-6)dx$

(e) $\frac{1}{x^2-9} = \frac{A}{x+3} + \frac{B}{x-3}$

$1 = A(x-3) + B(x+3)$

Let $x=3$: $1 = A(0) + B(6) \rightarrow B = \frac{1}{6}$

Let $x=-3$: $1 = A(-6) + B(0) \rightarrow A = \frac{-1}{6}$

$$\int \frac{dx}{x^2-9} = \frac{1}{6}\int \frac{1}{x-3}dx - \frac{1}{6}\int \frac{1}{x+3}dx$$

$$= \frac{1}{6}\ln|x-3| - \frac{1}{6}\ln|x+3|$$

$$= \frac{1}{6}\ln\frac{|x-3|}{|x+3|} + C$$

(9) $$\frac{2}{2x^2+3x-2} = \frac{A}{(2x-1)} + \frac{B}{(x+2)}$$

$$2 = A(x+2) + B(2x-1)$$

Let $x=-2$: $2 = A(0) + B(-5) \longrightarrow B = -\frac{2}{5}$

Let $x=\frac{1}{2}$: $2 = A\left(\frac{5}{2}\right) + B(0) \longrightarrow A = \frac{4}{5}$

$$\frac{2}{5}\int \frac{\overset{du}{\cancel{2\,dx}}}{\underset{u}{\cancel{2x-1}}} - \frac{2}{5}\int \frac{dx}{x+2}$$

$u = 2x-1$ $\qquad$ $u = x+2$

$du = 2dx$ $\qquad$ $du = dx$

$$\frac{2}{5}\ln|2x-1| - \frac{2}{5}\ln|x+2| + C$$

(h) $$\frac{3x}{2x^2+3x-2} = \frac{A}{2x-1} + \frac{B}{x+2}$$

$$3x = A(x+2) + B(2x-1)$$

Let $x=-2$: $-6 = A(0) + B(-5) \longrightarrow B = \frac{6}{5}$

Let $x=\frac{1}{2}$: $\frac{3}{2} = A\left(\frac{5}{2}\right) + B(0) \longrightarrow A = \frac{3}{5}$

$$\frac{1}{2}\cdot\frac{3}{5}\int \frac{\cancel{2dx}\ du}{\cancel{2x-1}\ u} + \frac{6}{5}\int\frac{dx}{x+2}$$

$u = 2x-1$

$du = 2dx$

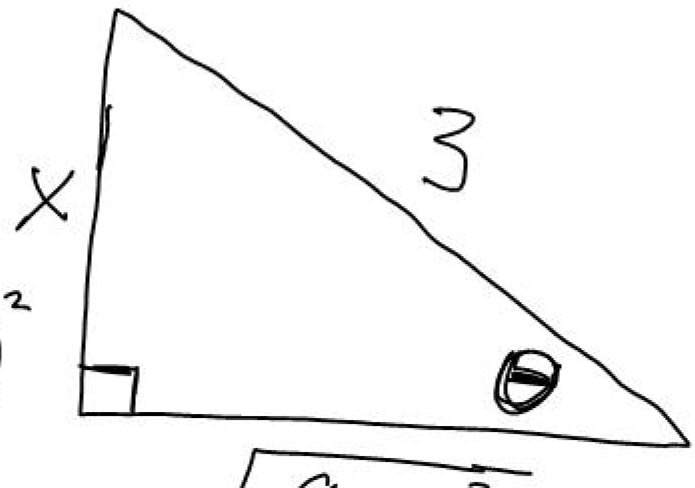
$$\frac{3}{10}\ln|2x-1| + \frac{6}{5}\ln|x+2| + C$$

Trig Substitution

$x^2 + a^2$ or $x^2 - a^2$ or $\underline{a^2 - x^2}$

Ex $\displaystyle\int \frac{dx}{x^2\sqrt{9-x^2}}$

$(hyp)^2 - (leg)^2$



$$= \int \frac{3\cos\theta\, d\theta}{9\sin^2\theta\, \underbrace{\sqrt{9-9\sin^2\theta}}_{\text{trig id}}}$$

$\sqrt{9(1-\sin^2\theta)}$

$\sqrt{9\cos^2\theta}$

$3\cos\theta$

$$\frac{x}{3} = \sin\theta$$

$$x = 3\sin\theta$$

$$\frac{dx}{\cancel{d\theta}} = 3\cos\theta\, d\theta$$

$$\boxed{\begin{aligned} &\sin^2\theta + \cos^2\theta = 1 \\ &1 + \tan^2\theta = \sec^2\theta \end{aligned}}$$

$$= \int \frac{\cancel{3\cos\theta}\, d\theta}{9\sin^2\theta \cdot \cancel{3\cos\theta}}$$

$$= \frac{1}{9}\int \csc^2\theta \, d\theta$$

$$= -\frac{1}{9}\cot\theta + C$$

the antiderivative

$$= -\frac{1}{9}\,\frac{\sqrt{9-x^2}}{x} + C$$

$\cot\theta$ is $\dfrac{\text{adj}}{\text{opp}}$

A note on antiderivatives w/ trig sub

$$\int \sin^2\theta \, d\theta = \int \frac{1}{2}\, d\theta - \frac{1}{2}\cdot\frac{1}{2}\int 2\cos 2\theta \, d\theta$$

$u = 2\theta$

$du = 2\,d\theta$

$$= \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C$$

$$\cos^2\theta + \sin^2\theta = 1$$

subtract

$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$2\sin^2\theta = 1 - \cos 2\theta$$

$$\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$$

$$\int \sin^2 x \, dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$$

$$\int \cos^2 x \, dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$$