#1. (s) $\int \frac{e^x}{e^x+2}\,dx = \int \frac{du}{u}$

$u = e^x + 2$

$du = e^x\,dx$

$= \ln|u| + C = \ln(e^x+2) + C$

(t) $-\int \frac{-\sin x}{\cos x + 2}\,dx = -\int \frac{du}{u}$

$u = \cos x + 2$

$du = -\sin x\,dx$

$= -\ln(\cos x + 2) + C$

2. $2\int_1^4 \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx = 2\int_e^{e^2} du = 2\left[u\right]_e^{e^2}$

$= 2e^2 - 2e$

$u = e^{\sqrt{x}}$

$du = e^{\sqrt{x}} \cdot \frac{dx}{2\sqrt{x}}$

The Product Rule for Antiderivatives
(sort of)

$$\frac{d}{dx}\left[u \cdot v\right] = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

$$\int d(uv) = \int u \cdot dv + \int v \cdot du$$

$$\int u \cdot dv = \int d(uv) - \int v \cdot du$$

$$\boxed{\int u \cdot dv = u \cdot v - \int v \cdot du}$$

The Parts Rule

Integration by Parts

$$\int u \cdot dv = u \cdot v - \int v \cdot du$$

Ex $$\int \underline{x} \cdot \underline{\sin x \; dx} = x(-\cos x) + \int + \cos x \, dx$$

$\underline{u} = x$ $\quad v = -\cos x$

$du = dx$ $\quad \int \underline{dv} = \int \sin x \, dx$

$$= \underline{-x \cdot \cos x + \sin x + C}$$

check $\frac{d}{dx}\left[-x \cdot \cos x + \sin x\right]$

$$= -x \cdot -\sin x + \cancel{\cos x \cdot (-1)} + \cancel{\cos x}$$

$$= x \cdot \sin x$$

Parts again

Ex. $\int x^2 \cdot e^x\,dx = x^2 e^x - \int 2x \cdot e^x\,dx$

$u = x^2$ $\qquad$ $v = e^x$ $\qquad$ $u = 2x$

$du = 2x\,dx$ $\qquad$ $dv = e^x\,dx$ $\qquad$ $du = 2\,dx$

$$\int x^2 e^x\,dx = x^2 e^x - \left(2xe^x - \int 2e^x\,dx\right)$$

$$= \underline{\underline{x^2 \cdot e^x - 2xe^x + 2e^x + C}}$$

check

$$x^2 \cdot e^x + \cancel{e^x \cdot 2x} - (\cancel{2xe^x} + \cancel{e^x \cdot 2}) + \cancel{2e^x}$$

$$\int x^3 \cdot \ln x \, dx = \frac{1}{4} x^4 \ln x - \int \frac{1}{4} x^4 \cdot \frac{1}{x} dx$$

$u = \ln x \qquad v = \frac{1}{4} x^4$

$du = \frac{1}{x} dx \quad \int dv = \int x^3 \, dx$

$$= \frac{x^4}{4} \ln x - \int \frac{1}{4} x^3 dx$$

$$= \boxed{\frac{x^4}{4} \cdot \ln x - \frac{x^4}{16} + C}$$

<u>Rule of Thumb</u> for when to use parts

$$\underbrace{\int x^n \sin x \, dx, \quad \int x^n \cos x \, dx, \quad \int x^n \cdot e^x dx}_{u = x^n}$$

$\int x^n \cdot \ln x \, dx$

$u = \ln x$

$\int \sin^{-1} x \, dx$

$\int \tan^{-1} x \, dx$

$\int e^x \cdot \sin x \, dx$

$\int e^x \cdot \cos x \, dx$

Ex. $\int \ln x \, dx = x \cdot \ln x - \int \cancel{x \cdot \frac{1}{x}} dx$

$u = \ln x \qquad v = x$

$du = \frac{1}{x} dx \quad \int dv = \int dx$

$$\boxed{\int \ln x \, dx = x \cdot \ln x - x + C}$$

$$\frac{d}{dx}\left[x^2\right] = 2x$$

$$\int x^2 \, dx = \frac{1}{3} x^3 + C$$

$$\int \tan^{-1} x \, dx = x \cdot \tan^{-1} x - \frac{1}{2} \int \frac{2x}{x^2+1} \, dx$$

$u = \tan^{-1} x \qquad v = x$

$du = \dfrac{dx}{x^2+1} \qquad \int dv = \int dx$

$u = x^2 + 1$

$du = 2x \, dx$

$-\dfrac{1}{2} \int \dfrac{1}{u} \, du$

$$\boxed{\int \tan^{-1} x \, dx = x \cdot \tan^{-1} x - \frac{1}{2} \ln\left(x^2+1\right) + C}$$

Shortcut: Tabular Integration

for $\int x^n \cdot \sin x, \cos x, \text{ or } e^x$, $n > 1$

$\int x^4 \cdot \cos x \, dx$

u	dv
x^4	$\cos x$
$(-1)\ 4x^3$	$\sin x$
$12x^2$	$-\cos x$
$(-1)\ 24x$	$-\sin x$
24	$\cos x$
0	$\sin x$

$= x^4 \sin x$
$+ 4x^3 \cos x$
$- 12x^2 \sin x$
$- 24x \cos x$
$+ 24 \sin x + C$