Antiderivatives

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$$

$$\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \tan x\,dx = -\ln|\cos x| + C$$
$$= \ln|\sec x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x \, dx = \ln|\csc x - \cot x| + C$$

$$\int \frac{1}{x^2+1} \, dx = \tan^{-1} x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C$$

$$\int \sec x \cdot \tan x \, dx = \sec x + C$$

$$\int \csc x \cdot \cot x \, dx = -\csc x + C$$

$$\int \sin^2 x \, dx =$$

$$\int \cos^2 x \, dx =$$

$$\int \tan^2 x \, dx = \int (\sec^2 x - 1)\, dx$$
$$= \tan x - x + C$$

$$\int \cot^2 x \, dx = \int (\csc^2 x - 1)\, dx$$
$$= -\cot x - x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

(k) $-1\int \frac{-1 \tan \frac{1}{x}}{x^2}\,dx = -\int \tan u \, du$

$= \ln\left|\cos \frac{1}{x}\right| + C$

$u = \frac{1}{x}$

$du = \frac{-1}{x^2}\,dx$

(p) $2\int \frac{\sec \sqrt{x}}{2\sqrt{x}}\,dx = 2\int \sec u \, du$

$u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2}x^{-1/2}\,dx = \frac{dx}{2\sqrt{x}}$

$= 2\ln\left|\sec\sqrt{x} + \tan\sqrt{x}\right| + C$

(q)

(9) $\int x\sqrt{(x-3)^4}\,dx = \int (u+3)u^4\,du$

$u = x-3 \rightarrow x = u+3$

$du = dx$

$= \int (u^5 + 3u^4)\,du = \frac{1}{6}u^6 + \frac{3}{5}u^5 + ($

$= \frac{1}{6}(x-3)^6 + \frac{3}{5}(x-3)^5 + C$

$\frac{d}{dx}$: $(x-3)^5 + 3(x-3)^4$

$(x-3)^4\left[x - \cancel{3} + \cancel{3}\right]$

(r) $\frac{1}{2}\int 2x^3\sqrt{x^2+1}\,dx$

$$u = x^2+1 \rightsquigarrow \underline{x^2 = u-1}$$

$$du = \underline{2x\,dx}$$

$$= \frac{1}{2}\int \underline{2x}\cdot x^2\sqrt{x^2+1}\,\underline{dx}$$

$$= \frac{1}{2}\int (u-1)u^{1/2}\,du$$

$$= \frac{1}{2}\int \left(u^{3/2} - u^{1/2}\right)du$$

$$= \frac{1}{5}u^{5/2} - \frac{1}{3}u^{3/2} + C$$

$$= \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C$$

The FTC and u-substitution

Ex. $\frac{1}{2}\int_0^1 2x(x^2+1)^4\,dx = \frac{1}{2}\int_1^2 u^4\,du$

$u = x^2+1$

$du = 2x\,dx$

$u = 0^2+1$

$u = 1^2+1$

$= \left[\frac{1}{10}u^5\right]_1^2$

$= \frac{1}{10}(2)^5 - \frac{1}{10}(1)^5$

$= \frac{31}{10}$

$\boxed{\text{HW } \#2}$