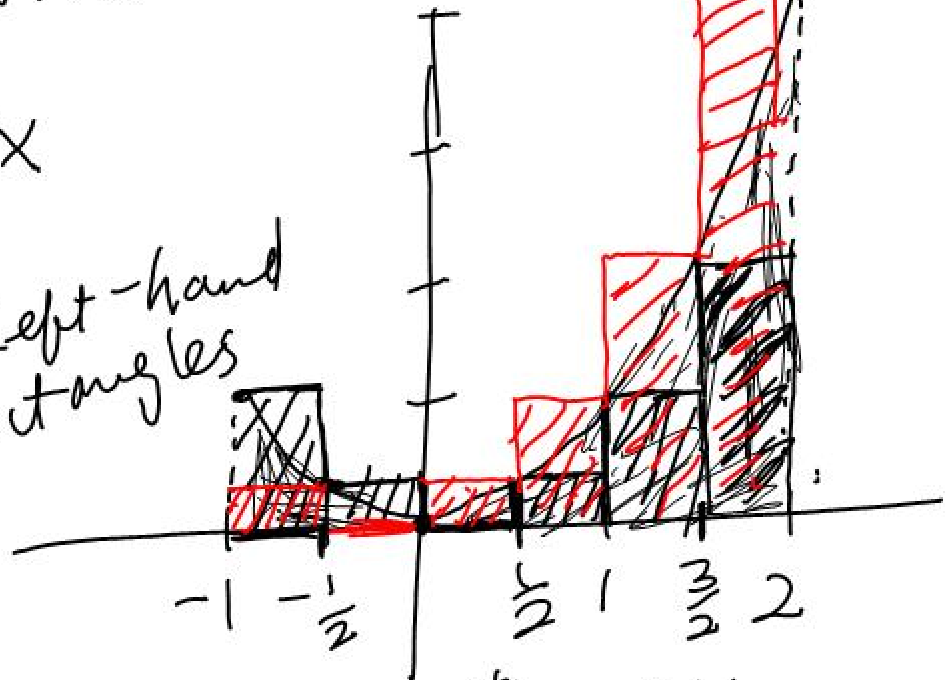
What about non-circular curves?

Ex

Left-hand rectangles



A Riemann Sum (Rectangles)

$$\int_{-1}^{2} x^2\,dx \approx \overset{\text{width}}{\frac{1}{2}}\left[\overbrace{1+\frac{1}{4}+0+\frac{1}{4}+1+\frac{9}{4}}^{\text{heights}}\right]$$

$$= \frac{19}{8} = 2.375$$

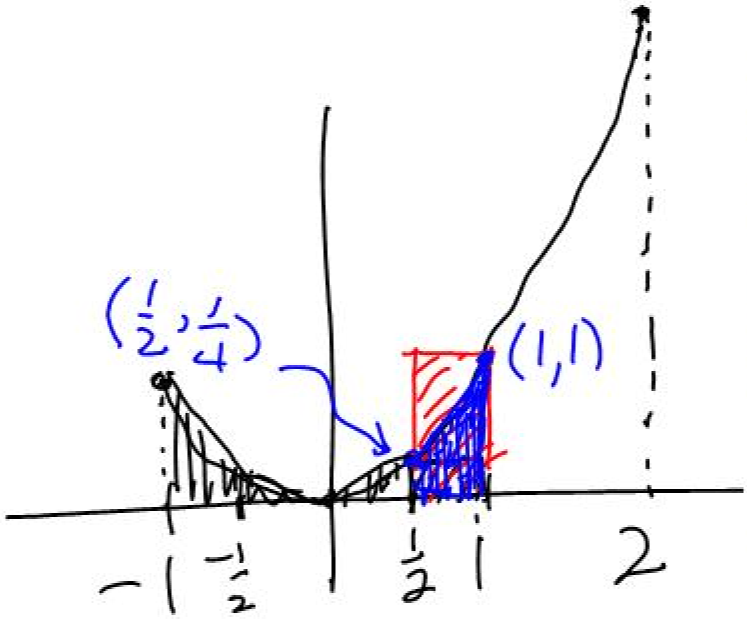
Right-hand Riemann Sum

$$\int_{-1}^{2} x^2\,dx \approx \frac{1}{2}\left[\frac{1}{4}+0+\frac{1}{4}+1+\frac{9}{4}+4\right] =$$

$$= 3{,}875$$

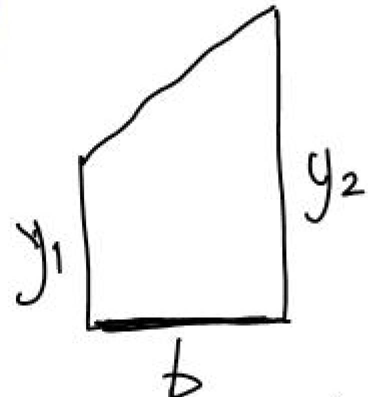
Best estimate so far: $\displaystyle\int_{-1}^{2} x^2\,dx \approx \frac{2{,}375+3{,}875}{2}$

$$= 3.125$$



Averaging left- + right-hand rectangles gives the same estimate as finding the area under <u>trapezoids</u>.

<u>The Trapezoid Method</u>



$$\text{area} = \frac{1}{2} b (y_1 + y_2)$$

trap#1 trap#2 ...

$$\int_{-1}^{2} x^2 \, dx \approx \frac{1}{2} \cdot \frac{1}{2} \Big[\left(1 + \frac{1}{4}\right) + \left(\frac{1}{4} + 0\right) + \left(0 + \frac{1}{4}\right) + \left(\frac{1}{4} + 1\right) + \left(1 + \frac{9}{4}\right) + \left(\frac{9}{4} + 4\right) \Big]$$

area formula for a trapezoid

base (Δx)

6 Trapezoids

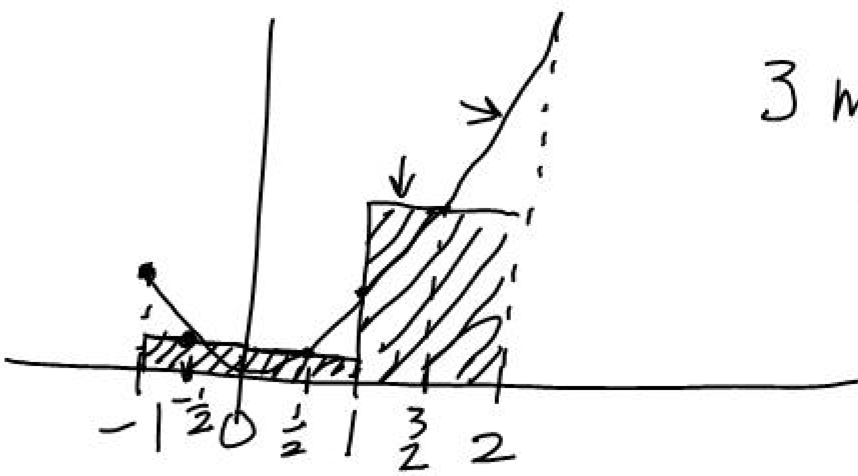
$$\left\{ \Delta x = \frac{b-a}{n} \right\}$$

$$\Delta x = \frac{2 - -1}{6} = \frac{1}{2}$$

$$\int_{-1}^{2} x^2\,dx \approx \frac{1}{2}\cdot\frac{1}{2}\left[1 + 2\left(\tfrac{1}{4}\right) + 2(0) + 2\left(\tfrac{1}{4}\right) + 2(1) + 2\left(\tfrac{9}{4}\right) + 4\right]$$

$$= \frac{1}{4}\left[\frac{25}{2}\right] = 3.125$$

The Midpoint Riemann Sum



3 midpoint rectangles

$$\Delta x = \frac{2 - -1}{3} = 1$$

$$\int_{-1}^{2} x^2\,dx \approx \underset{\Delta x}{1}\left[\left(-\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right)^2 + \left(\tfrac{3}{2}\right)^2\right] = 2.75$$

Right-hand sum

$$\int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \Delta x \left[f(a+\Delta x) + f(a+2\Delta x) + \cdots + f(b) \right]$$

HW Approximating Def. Int # 1–4,

5 abc