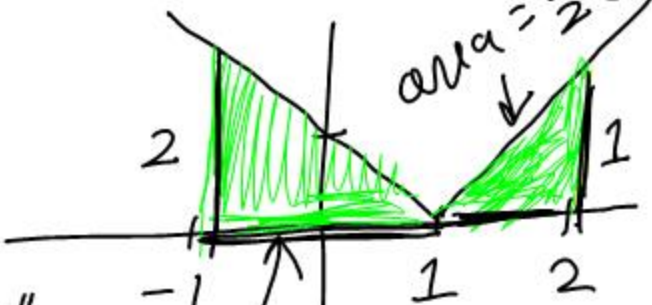
Definite Integrals

calculus
- differential calculus
- integral calculus
- series

Ex $\int_{-1}^{2} |x-1|\,dx = 2 + \frac{1}{2} = 2.5$

integral sign "S" = "sum"



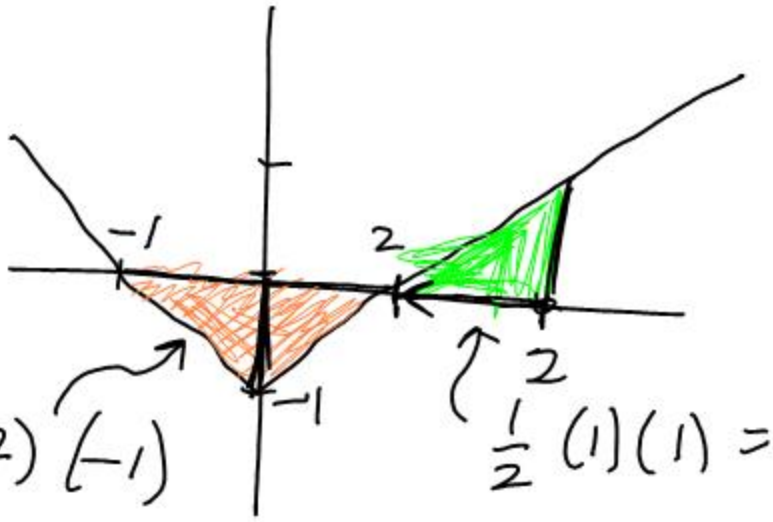
area = $\frac{1}{2}(2)(2) = 2$

$\frac{d}{dx}[x^2] = 2x$

↑ x is the independent variable

$\frac{d}{dt}[\pi r^2]$

Ex $\int_{-1}^{2} (|x| - 1)\, dx = -1 + \frac{1}{2} = \frac{-1}{2}$



"algebraic area"

area $= \frac{1}{2}(2)(-1)$ $\quad \frac{1}{2}(1)(1) = \frac{1}{2}$

$= -1$

Ex: $\int_{2}^{-1} (|x| - 1)\, dx = 1 - \frac{1}{2} = +\frac{1}{2}$

big $\Delta = \frac{1}{2}(-2)(-1) = +1$

small $\Delta = \frac{1}{2}(-1)(1) = -\frac{1}{2}$

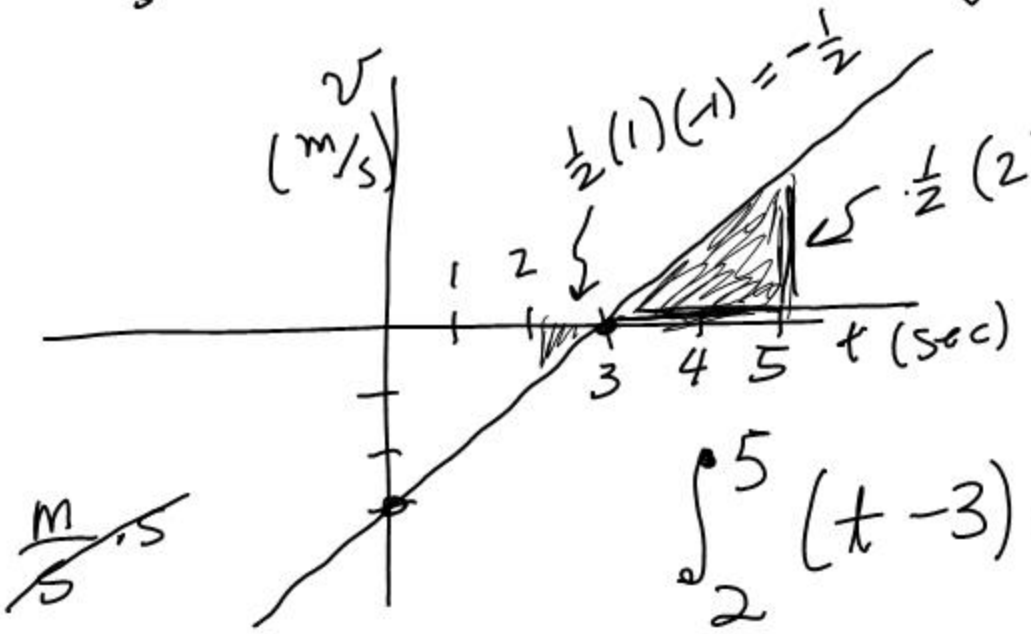
general rule:

$$\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$$

Ex. (Physics)

$v(t) = t - 3$ gives the velocity of a particle moving along a line

$\frac{m}{s}$ s

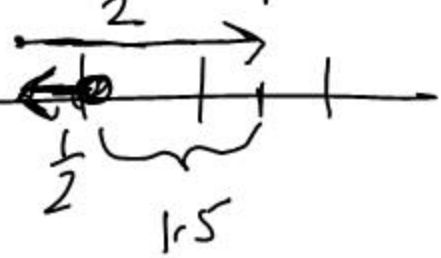


$\frac{m}{s}$ .5

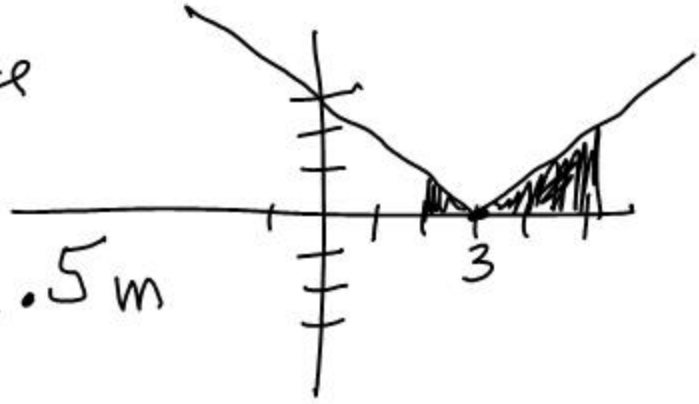
$$\int_2^5 (t-3)\,dt = \underline{1.5\ m}$$

↑ displacement

$$\int_{t_1}^{t_2} v(t)\,dt = \text{displacement}$$

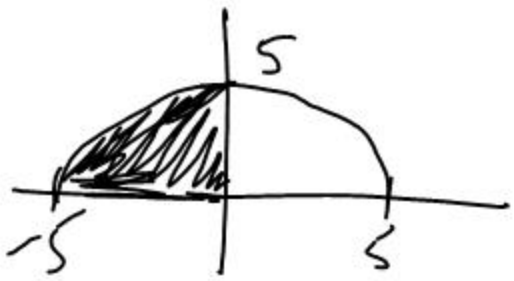


$$\int_{t_1}^{t_2} |v(t)|\,dt = \text{distance}$$



$$\int_2^5 |t-3|\,dt = 2.5\ m$$

Ex. $\int_0^{-5} \sqrt{25 - x^2}\, dx$



$= -\frac{25\pi}{4}$

$y = \sqrt{25 - x^2}$

$y^2 = 25 - x^2$

$x^2 + y^2 = 25$

#11. (a) $\int_0^4 h(x)\, dx = 2\pi$

(c) $\int_4^7 h(x)\, dx = -2$

(e) $\int_{-7}^{-5} h(x)\, dx = 0$

#13

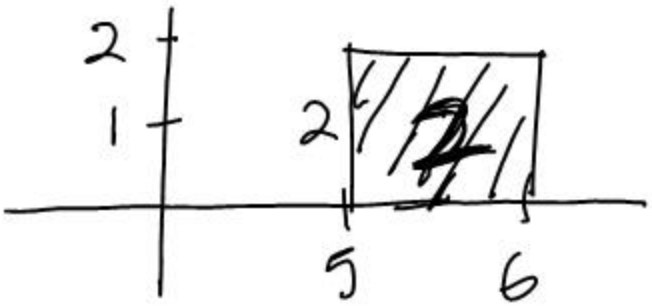
(b)

$G(3) = \int_3^0 f(t)\, dt$

$= -\left(\frac{9\pi}{4} + 3\right)$

14(c) $$\int_5^6 [2 + f(x)]\,dx$$

$$= \int_5^6 2\,dx + \int_5^6 f(x)\,dx = 2$$

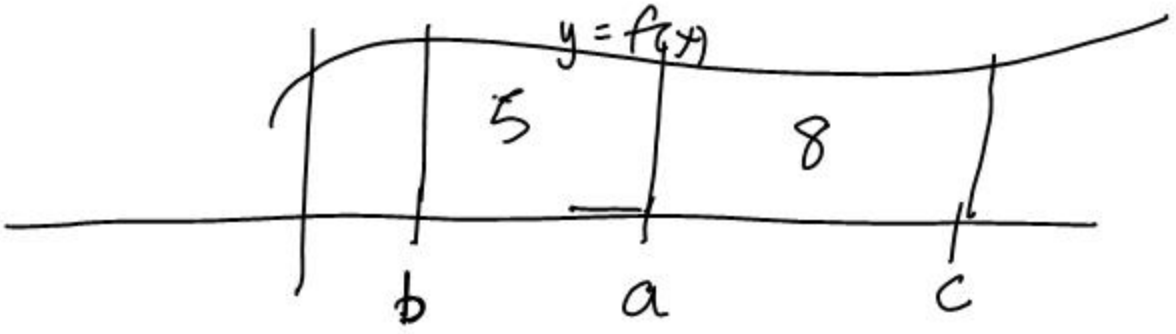


14(f) $$\int_0^3 [f(x) - 2x]\,dx$$

$$= \int_0^3 f(x)\,dx - 2\int_0^3 x\,dx$$

more general rules:

- $\int_a^b \left(f(x) \pm g(x)\right) dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
- $\int_a^b c \cdot f(x)\,dx = c \int_a^b f(x)\,dx$
- $\int_a^a f(x)\,dx = 0$
- $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$



$\int_a^b f(x)dx = -5 \quad \int_b^c f(x) = 13 \quad \int_a^c f(x) = 8$

$-5 + 13 = 8$

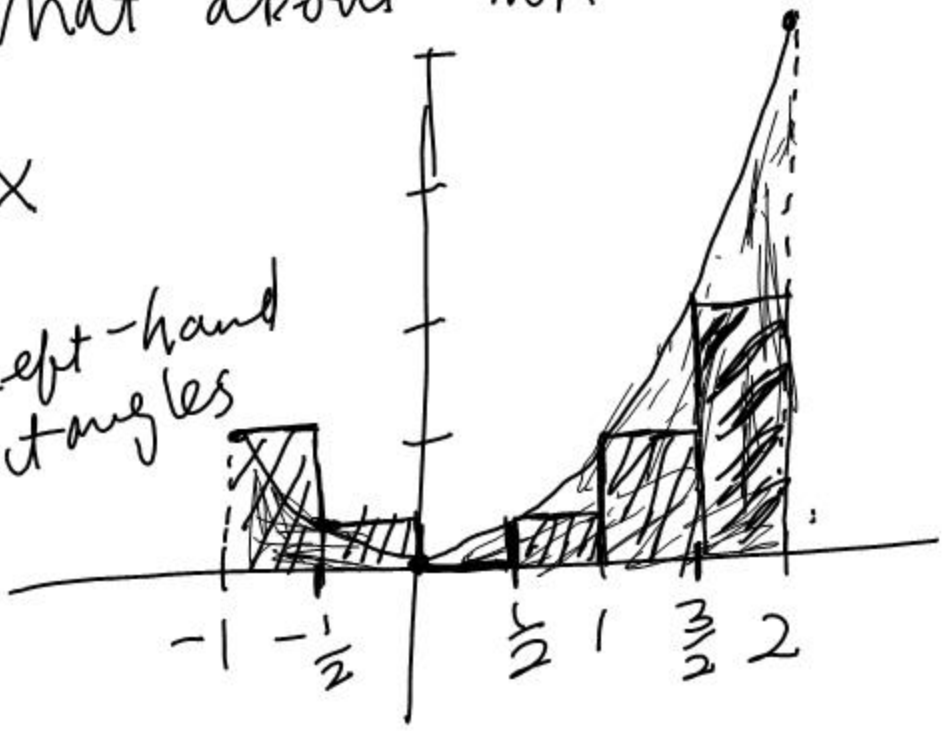
#3. $\int_2^4 [3f(x) - 2g(x)]\,dx$

$3\int_2^4 f(x)\,dx - 2\int_2^4 g(x)\,dx$

$3(10) - 2(5) = 20$

What about non-circular curves?

Ex



$$\int_{-1}^{2} x^2\,dx \approx \frac{1}{2}\left[1+\frac{1}{4}+0+\frac{1}{4}+1+\frac{9}{4}\right]$$

$$= \frac{19}{8} = 2.38$$