$\boxed{\#5}$ $y = x^4 - 2x^2$

$y' = 4x^3 - 4x = 0$ ← derivative

$4x(x^2 - 1) = 0$ factor

$4x(x+1)(x-1) = 0$

$x = 0, \pm 1$ ← critical values

$f'(x)$: $\quad - \;\vee\; + \;\cap\; - \;\vee\; +$

$\qquad\qquad -1 \qquad 0 \qquad 1$

max at $x = 0$ $(0,0)$

min at $x = \pm 1$ $(1,-1)$

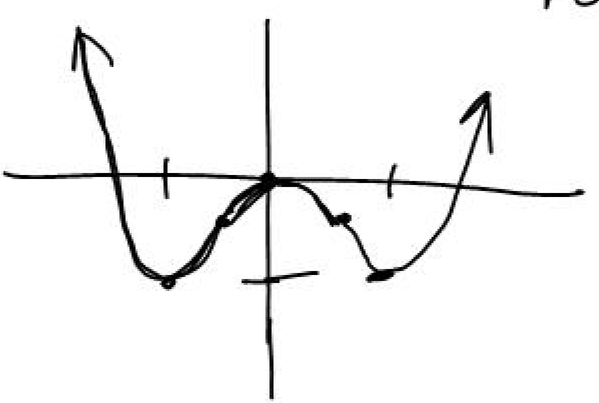
$y'' = 12x^2 - 4 = 0$

$4(3x^2 - 1) = 0$

$x = \pm \frac{1}{\sqrt{3}}$

$f''(x)$: $\quad + \qquad - \qquad +$

$\qquad\qquad -\frac{1}{\sqrt{3}} \qquad \frac{1}{\sqrt{3}}$



(b) $y = 2xe^x$

$\lim_{x\to-\infty} 2xe^x = \lim_{x\to-\infty} \dfrac{2e^x}{\frac{1}{x}}$

$-\infty \cdot 0$ $\quad \frac{0}{0}$

$= \lim_{x\to-\infty} \dfrac{2e^x}{-\frac{1}{x^2}}$

$y' = 2x\cdot e^x + e^x \cdot 2$

$= 2e^x(x+1) = 0 \Rightarrow x = -1$

y': $\overset{-}{\longleftarrow} \underset{-1}{|} \overset{+}{\longrightarrow}$ min. at $x = -1$

$\left(-1, \frac{-2}{e}\right)$

$-.74$

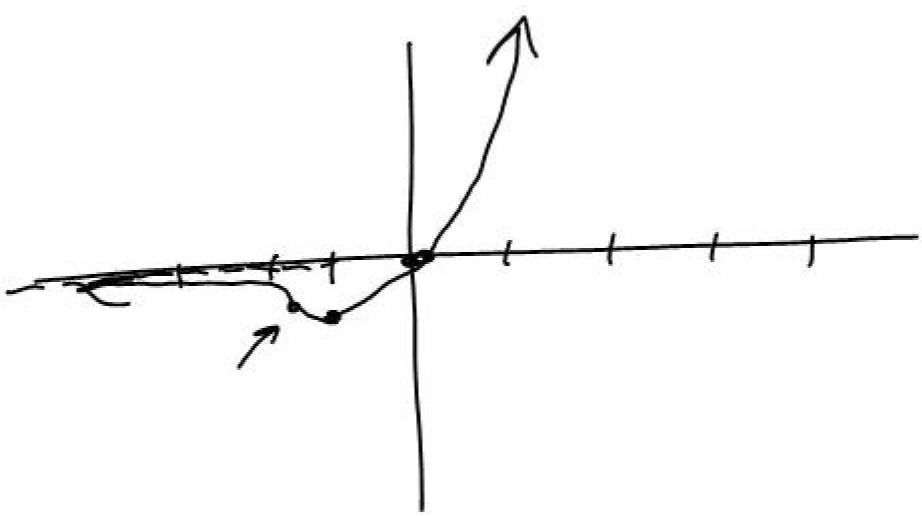
$y'' = 2xe^x \cancel{+ e^x \cdot 2 + 2e^x}$ $+3e^x$

$= e^x(2x+3) = 0 \Rightarrow x = -3/2$

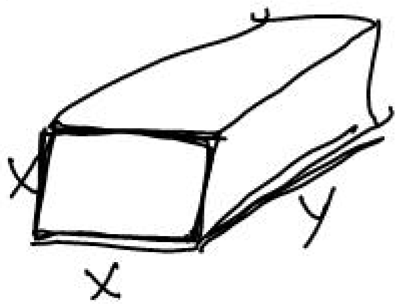
y'': $\overset{-}{\longleftarrow} \underset{-3/2}{|} \overset{+}{\longrightarrow}$ flex pt: $x = -3/2$

$\left(-\frac{3}{2}, \frac{-3}{e^{3/2}}\right)$

-0.67

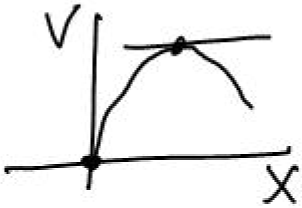


#4



$$4x + y = 64 \Rightarrow y = 64 - 4x$$

$$V = x^2 y \leftarrow \text{maximum}$$



$$V = x^2(64-4x) = 64x^2 - 4x^3$$

$$V' = 128x - 12x^2 = 0$$

$$4x(32 - 3x) = 0$$

$$\cancel{x = 0} \qquad \boxed{x = \frac{32}{3} \text{ in.}}$$

#2a

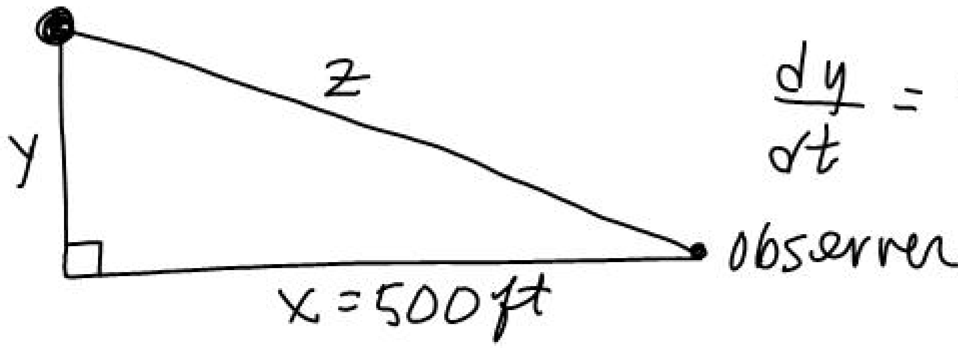
$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$100 = 4\pi (5)^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{100}{100\pi} = \frac{1}{\pi} \text{ ft/min}$$

#1



$$\frac{dy}{dt} = 4 \text{ ft/sec}$$

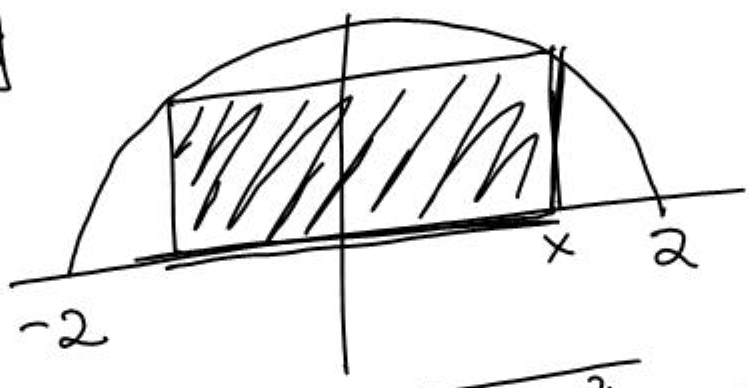
(a) $y = 1200$ ft $\Rightarrow$ $z = 1300$ ft

(b) $y^2 + 500^2 = z^2$

(c) $\cancel{2}y \frac{dy}{dt} = \cancel{2}z \frac{dz}{dt}$

$$\left.\frac{dz}{dt}\right|_{y=1200} = \frac{1200}{1300} \cdot 4 = \frac{48}{13} \text{ ft/sec}$$

#3



$$A = 2x\sqrt{4-x^2}$$

$$A' = 2\cancel{x} \cdot \frac{1}{\cancel{2}}(4-x^2)^{-1/2}(-2x) + (4-x^2)^{1/2} \cdot 2 = 0$$

$$2(4-x^2)^{-\frac{1}{2}}\left[-x^2 + (4-x^2)\right] = 0$$

$$A' = 2\underline{(4-x^2)^{-1/2}}\,\underline{(4-2x^2)} = 0$$

$$-2x^2 = -4$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

#7a

$$\lim_{x\to\infty}\left(1-\frac{3}{x}\right)^x = e^{-3} = \frac{1}{e^3}$$

1^∞

$\boxed{\infty \cdot 0}$

New problem: $\lim_{x\to\infty} \ln\left(1-\frac{3}{x}\right)^x = \lim_{x\to\infty} \cancel{x}\cdot\ln\left(1-\frac{3}{x}\right)$

$$= \lim_{x\to\infty} \frac{\ln\left(1-\frac{3}{x}\right)}{\frac{1}{x}} \overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{1-3/x}\cdot\frac{3}{\cancel{x^2}}}{\frac{-1}{\cancel{x^2}}}$$

$1 \quad 3 \quad -1$

$\boxed{\frac{0}{0}}$

$-3x^{-1}$

x^{-1}

$$= -3$$

(b) $$\lim_{x\to 0}\frac{\sin x - x}{x^3} = \lim_{x\to 0}\frac{\cos x - 1}{3x^2}$$

$\frac{0}{0} \qquad \frac{0}{0}$

$$= \lim_{x\to 0}\frac{-\sin x}{6x} = \lim_{x\to 0}\frac{-\cos x}{6} = \frac{-1}{6}$$

$\frac{0}{0}$

n	$f^{(n)}(x)$	$f^{(n)}(0)$	$\frac{f^{(n)}(0)}{n!}$
0	$(x+4)^{1/2}$	$2 \div 0!$	2
1	$\frac{1}{2}(x+4)^{-1/2}$	$\frac{1}{4} \div 1!$	$\frac{1}{4}$
2	$-\frac{1}{4}(x+4)^{-3/2}$	$-\frac{1}{32} \div 2!$	$-\frac{1}{64}$
3	$\frac{3}{8}(x+4)^{-5/2}$	$\frac{3}{8}\cdot\frac{1}{32} = \frac{1\cancel{3}}{256}$ $\div \cancel{3}\cdot 2$	$\frac{1}{512}$

$$\boxed{f(x) \approx 2 + \frac{1}{4}x - \frac{1}{64}x^2 + \frac{1}{512}x^3}$$

$$x^2\sqrt{x+4} \approx 2x^2 + \frac{1}{4}x^3 - \frac{1}{64}x^4 + \frac{1}{512}x^5$$

#26

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$\left.\frac{dA}{dt}\right|_{r=5} = 8\cancel{\pi}(5)\left(\frac{1}{\cancel{\pi}}\right) = 40 \text{ ft}^2/\text{min}$$

$\boxed{\#1}$

$$\lim_{x\to 0^+} 4x \cdot \ln\frac{1}{x} = \lim_{x\to 0^+} \frac{4\ln\frac{1}{x}}{\frac{1}{x}}$$

$\boxed{0\cdot\infty}$ $\boxed{\frac{\infty}{\infty}}$

$$= \lim_{x\to 0^+} \frac{4\cdot x\cdot \cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}} = 0$$

$\boxed{\#6}$

$$\lim_{x\to 0}\left(\frac{3}{x} - \frac{3}{e^x-1}\right)$$

$\boxed{\infty-\infty}$

$$= \lim_{x\to 0}\left(\frac{3(e^x-1)}{x(e^x-1)} - \frac{3x}{(e^x-1)x}\right)$$

$$= \lim_{x\to 0}\frac{3e^x-3x-3}{xe^x-x}$$

$\frac{0}{0}$

$$= \lim_{x\to 0}\frac{3e^x-3}{xe^x+e^x-1}$$

$\frac{0}{0}$

$\frac{0}{0}$

$\frac{\infty}{\infty}$

$\infty-\infty$

1^∞

0^0

$\infty\cdot 0$

$$= \lim_{x \to 0} \frac{3e^x}{\cancel{xe^x} + e^x + e^x} = \frac{3}{2}$$