#4 $\lim\limits_{x\to 0^+} \dfrac{e^x-(1-x)}{x^3} = \lim\limits_{x\to 0^+} \dfrac{e^x+1}{3x^2}$

$= \dfrac{1+1}{3(0)} = \infty$

#7. $\lim\limits_{x\to 1} \dfrac{\frac{1}{1+x^2}}{1} = \dfrac{1}{2}$

x^{-1}

#10 $\lim\limits_{x\to\infty} \dfrac{\tan\frac{1}{x}}{\frac{1}{x}} = \lim\limits_{x\to\infty} \dfrac{\sec^2\left(\frac{1}{x}\right)\cdot\cancel{\left(\frac{-1}{x^2}\right)}}{\cancel{\frac{-1}{x^2}}}$

$\frac{0}{0}$

$= 1$

#12 $\lim\limits_{x\to 0^+} \dfrac{\ln\left(e^x+x\right)}{x} = \lim\limits_{x\to 0^+} \dfrac{\frac{e^x+1}{e^x+x}}{1}$

$\frac{0}{0}$

$= \dfrac{2}{1} = 2$

$$\boxed{\lim_{x\to 0^+} \left(e^x+x\right)^{\frac{1}{x}} = e^2}$$

$\boxed{\#18}$ $\lim_{x\to\infty}\left(\sqrt{x^2-x}-x\right)$

$$\frac{1{,}000{,}000{,}000{,}000}{1000000}$$

$\boxed{\infty-\infty}$

indeterminate form

$$= \lim_{x\to\infty}\left(\frac{\sqrt{x^2-x}-x}{1}\right)\left(\frac{\sqrt{x^2-x}+x}{\sqrt{x^2-x}+x}\right)$$

$$= \lim_{x\to\infty}\frac{\cancel{x^2}-x-\cancel{x^2}}{\sqrt{x^2-x}+x}$$

$\frac{-\infty}{\infty}$

$$= \lim_{x\to\infty}\frac{-1}{\frac{1}{2}(x^2-x)^{-1/2}(2x-1)+1}$$

$$= \lim_{x\to\infty}\frac{-1}{\frac{2x-1+2\sqrt{x^2-x}}{2\sqrt{x^2-x}}}$$

$$\lim_{x\to\infty} \frac{-2\sqrt{x^2-x}}{2x-1+2\sqrt{x^2-x}}$$

$$\frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{-(x^2-x)^{-1/2}(2x-1)}{2+(x^2-x)^{-1/2}(2x-1)}$$

$$= \lim_{x\to\infty} \frac{\dfrac{-(2x-1)}{\sqrt{x^2-x}}}{\dfrac{2\sqrt{x^2-x}+2x-1}{\sqrt{x^2-x}}}$$

$$= \lim_{x\to\infty} \frac{-(2x-1)\sqrt{x^2-x}}{2(x^2-x)+(2x-1)\sqrt{x^2-x}}$$

$$\frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{-\overset{\infty}{(2x-1)}\,\frac{1}{2}\overset{\infty}{(x^2-x)^{\frac{1}{2}}}\,\overset{\infty}{(2x-1)} + \overset{\infty}{(x^2-x)^{\frac{1}{2}}}(-2)}{\underset{\infty}{(4x-2)} + \underset{2}{(2x-1)\frac{1}{2}(x^2-x)^{-\frac{1}{2}}(2x-1)} + \underset{\infty}{(x^2-x)^{\frac{1}{2}}\cdot 2}}$$

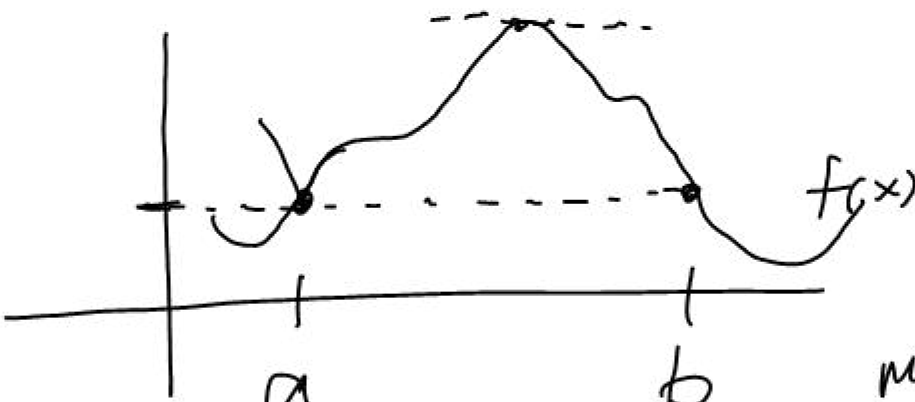
$$\frac{\infty}{\infty}$$

Zero exponents

$$5^0 = \frac{5^3}{5^3} = 5^{3-3} = 1$$

$$\cancel{0^0} = \cancel{\frac{0^3}{0^3}}$$

Rolle's Theorem



no holes
no jumps
no V.A.

If $f(x)$ is continuous on $[a, b]$

and differentiable on (a, b)

and $f(a) = f(b)$ $[= 0]$

no cusps

then there is at least one point $c \in (a, b)$ such that $f'(c) = 0$.

☆ The Mean Value Theorem (MVT)

If $f(x)$ is cont. on $[a, b]$

and diff. on (a, b),

then there exists at least one

$c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$