

$$\textcircled{\# 2} \quad x^2 \cdot e^x + e^x \cdot 2x$$

$$= x e^x (x + 2)$$

$$\textcircled{\# 3} \quad \frac{(x^3 + 1)(2x) - (x^2 + 1)(3x^2)}{(x^3 + 1)^2}$$

$$= \frac{2x^4 + 2x - 3x^4 - 3x^2}{(x^2 + 1)^2}$$

$$= \frac{-x^4 - 3x^2 + 2x}{(x^2 + 1)^2}$$

$$= \frac{-x(x^3 + 3x - 2)}{(x^3 + 1)^2}$$

(#4) $f(x) = \sin(\cos x)$ chain rule

$$f'(x) = \cos(\cos x) \cdot (-\sin x)$$

↑
NOT $\cos x \cdot \cos x$

(#5)

$$2 \tan x \cdot \sec^2 x$$

(#6) $f(x) = (x^2 + 3x + 1)^{1/2}$

$$f'(x) = \frac{1}{2} (x^2 + 3x + 1)^{-1/2} (2x + 3)$$

$$= \frac{2x + 3}{2\sqrt{x^2 + 3x + 1}}$$

$$\textcircled{\# 1.} \quad 2x - 5xy' - 5y + 4yy' = 0$$

$$y' = \frac{5y - 2x}{4y - 5x}$$

$$\# 2. \quad 2x + 2yy' = \frac{(x-y)(1+y') - (x+y)(1-y')}{(x-y)^2}$$

$$2x(x-y)^2 + \underline{2yy'(x-y)^2} = \textcircled{x} + xy' - y \textcircled{yy'}$$
$$\textcircled{-x} + xy' - y \textcircled{yy'}$$
$$= 2xy' - 2y$$

$$y' = \frac{-2y - 2x(x-y)^2}{2y(x-y)^2 - 2x}$$

$$= \frac{-y - x(x-y)^2}{y(x-y)^2 - 2x}$$

$$\#3 \quad \underline{x \cdot \cos y \cdot y'} + \sin y \cdot y \sin x + \underline{\cos x \cdot y'}$$

$$= \underline{2yy'}$$

$$y' = \frac{y \sin x - \sin y}{x \cos y + \cos x - 2y}$$

$$\#4 \quad \underline{e^y \cdot y'} - \underline{xg' - y} = \underline{x 2yy'} + y^2$$

$$y' = \frac{y^2 + y}{e^y - x - 2xy}$$

Other derivative formulas

- If $y = \ln x$, find $\frac{dy}{dx}$.
exponentiate to get

$$e^y = e^{\ln x}$$

$$e^y = x \leftarrow \text{Now, differentiate implicitly}$$

$$\frac{d}{dx} [e^y] = \frac{d}{dx} [x]$$

$$e^y \cdot y' = 1$$

$$y' = \frac{1}{e^y} = \frac{1}{e^{\ln x}}$$

$$\boxed{y' = \frac{1}{x}}$$

Ex. $f(x) = \ln(x^2 + x)$ Find $f'(x)$

$$f'(x) = \frac{1}{x^2 + x} \cdot (2x + 1) = \frac{2x + 1}{x^2 + x}$$

• If $y = \sin^{-1} x$, find $\frac{dy}{dx}$

Take the sine of both sides

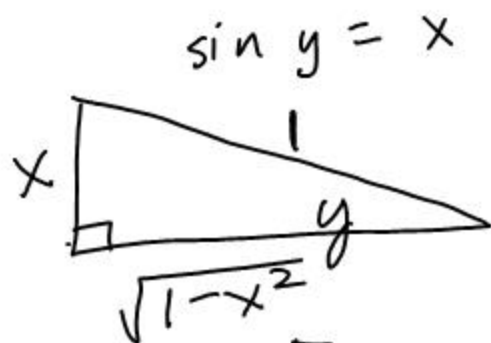
$\sin y = x$ differentiate implicitly

$$\frac{d}{dx} [\sin y] = \frac{d}{dx} [x]$$

$$\cos y \cdot y' = 1$$

$$y' = \frac{1}{\cos y}$$

$$y' = \frac{1}{\sqrt{1-x^2}}$$



Pythag.
Thm.

$$y = \sqrt{1-x^2}$$
$$y^2 = 1-x^2$$
$$x^2 + y^2 = 1$$

• If $y = \cos^{-1} x$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

$$\bullet \text{ If } y = \tan^{-1} x$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

Ex. $f(x) = \sin^{-1}\left(\frac{1}{x}\right)$ Find $f'(x)$

$$f'(x) = \frac{1}{\sqrt{1 - \left(\frac{1}{x}\right)^2}} \cdot \frac{-1}{x^2}$$

↑
deriv of $\frac{1}{x}$

$$= \frac{1}{\sqrt{\frac{x^2 - 1}{x^2}}} \cdot \frac{-1}{x^2}$$

$$= \frac{1}{\frac{\sqrt{x^2 - 1}}{|x|}} \cdot \frac{-1}{x^2}$$

$$= \frac{-|x|}{x^2 \sqrt{x^2 - 1}}$$

HW
implicit
5, 6
logs + mv.
1-6

$$\sqrt{(-5)^2} = \sqrt{25} = 5$$

$$y = \sqrt{x}$$

