"Big 5" questions

#7. $r = \cos^3\theta = (\underline{\cos\theta})^3$

$$\frac{dr}{d\theta} = 3(\cos\theta)^2 \cdot (-\sin\theta)$$

$$= -3\cos^2\theta \sin\theta$$

#8 $r = \cos(\theta^3)$

$$\frac{dr}{d\theta} = -\sin(\theta^3) \cdot 3\theta^2$$

#14 $A(y) = y^2 \cdot \underline{\sec^2 y}$

$$A'(y) = y^2 \cdot \overline{2\sec y \cdot \sec y \tan y}$$

$$+ \sec^2 y \cdot 2y$$

$$= 2y \cdot \sec^2 y \,(y \tan y + 1)$$

#16. $x(t) = \dfrac{1}{\sqrt{t^2+t+1}} = (t^2+t+1)^{-\frac{1}{2}}$

$$x'(t) = \frac{dx}{dt} = -\frac{1}{2}(t^2+t+1)^{-3/2}(2t+1)$$

$$= \frac{-(2t+1)}{2(t^2+t+1)^{3/2}}$$

#15 $f(z) = \frac{\pi}{2} \cdot e^{-\frac{1}{2}z^2}$

$$f'(z) = \frac{\pi}{2} \cdot e^{-\frac{1}{2}z^2} \cdot (-z)$$

#17. $y(t) = \tan(e^{2t})$

$$y'(t) = \frac{dy}{dt} = \sec^2(e^{2t}) \cdot e^{2t} \cdot 2$$

\# 19 $y(t) = \sin t \cdot e^{\sin t}$

$$y'(t) = \frac{dy}{dt} = \sin t \cdot e^{\sin t} \cdot \cos t + e^{\sin t} \cdot \cos t$$

$$= \underbrace{e^{\sin t}}_{\text{never zero}} \cdot \cos t \, (\sin t + 1)$$

\# 18 $f(x) = \sec^2 x - \tan^2 x = 1$

$f'(x) = 0$

$1 + \tan^2 \theta = \sec^2 \theta$

$\sec^2\theta - \tan^2 \theta = 1$

\# 22 point $(0, 1)$ $f'(t) = e^t$

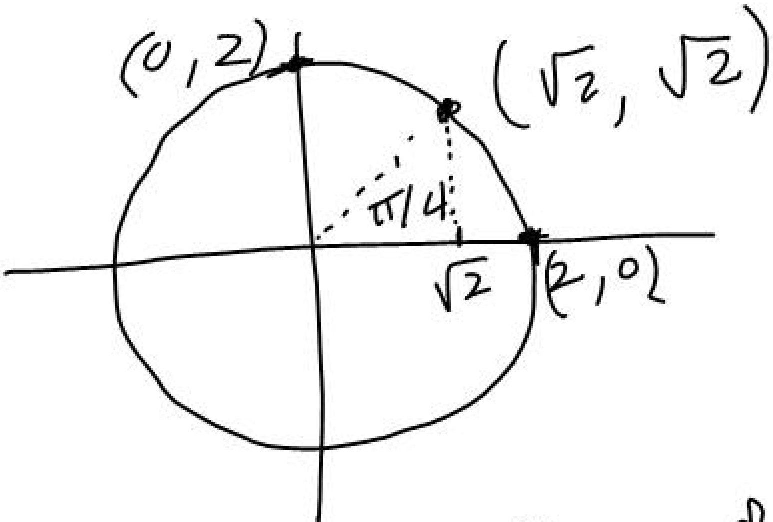
slope $f'(0) = 1$ $y - y_0 = m(x - x_0)$

tangent line: $y - 1 = 1(x - 0)$

$$\boxed{y = x + 1}$$

Implicit Differentiation

Circle: $x^2 + y^2 = 4$



$$\frac{d}{dx}\left[x^2 + y^2\right] = \frac{d}{dx}\left[4\right]$$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

derivative of the y
(the inner function)

$$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$$

$$\left.\frac{dy}{dx}\right|_{(\sqrt{2}, \sqrt{2})} = -\frac{\sqrt{2}}{\sqrt{2}} = -1$$

Ex. Find $\frac{dy}{dx}$ for $x^2 + 2xy + 3y^2 = 1$

$B^2 - 4AC = 2^2 - 4(1)(3) < 0$

$\Rightarrow$ ellipse

side note:

If $Ax^2 + Bxy + Cy^2 = D$

is a conic section,

then $B^2 - 4AC < 0 \rightarrow$ ellipse

$B^2 - 4AC = 0 \rightarrow$ parabola

$B^2 - 4AC > 0 \rightarrow$ hyperbola

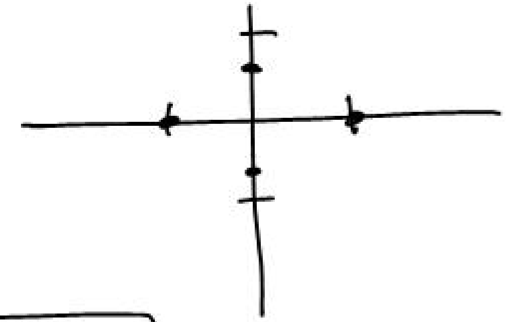
$$\frac{d}{dx}\left[x^2 + 2x \cdot y + 3y^2\right] = \frac{d}{dx}[1]$$

$$2x + \left(2x\frac{dy}{dx} + y \cdot 2\right) + 6y \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2x + 6y) = -2x - 2y$$

$$\boxed{\frac{dy}{dx} = \frac{-2x-2y}{2x+6y}} = \frac{-x-y}{x+3y}$$

$$x^2 + 2xy + 3y^2 = 1$$



a) Find the intercepts

<u>x-int</u> $\quad x^2 = 1 \Rightarrow \boxed{x = \pm 1}$

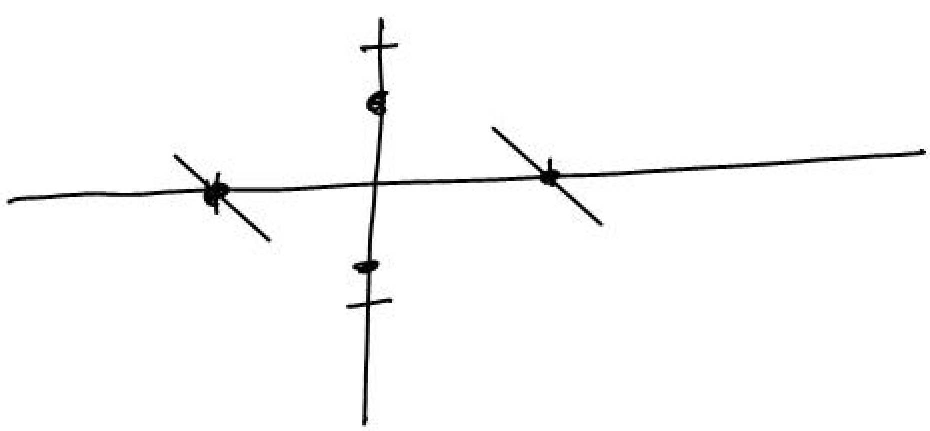
<u>y-int</u> $\quad 3y^2 = 1 \Rightarrow y^2 = \frac{1}{3} \Rightarrow y = \pm \frac{1}{\sqrt{3}}$

b) Find the tangent line slope at the x-intercepts.

$-2x -$

$$(1, 0) \rightarrow \left.\frac{dy}{dx}\right|_{(1,0)} = \frac{-1 - 0}{1 - 3(0)} = -1$$

$$(-1, 0) \rightarrow \left.\frac{dy}{dx}\right|_{(-1,0)} = \frac{-(-1) - 0}{-1 + 3(0)} = -1$$



Ex. For $x^3 - x^2 y - y^4 = x$

Find $\frac{dy}{dx}$

$$3x^2 - \left(x^2 \frac{dy}{dx} + y \cdot 2x\right) - 4y^3 \cdot \frac{dy}{dx} = 1$$

$$3x^2 - x^2 \frac{dy}{dx} - 2xy - 4y^3 \frac{dy}{dx} = 1$$

$$\frac{dy}{dx}\left(-x^2 - 4y^3\right) = 1 - 3x^2 + 2xy$$

$$\boxed{\frac{dy}{dx} = \frac{1 - 3x^2 + 2xy}{-x^2 - 4y^3}}$$

Ex. Find y' if $x \cdot \sin y = y \cdot \cos x$

$$x \cdot \cos y \cdot y' + \sin y = y \cdot (-\sin x) + \cos x \cdot y'$$

$$y'(x \cos y - \cos x) = -y \sin x - \sin y$$

$$\boxed{y' = \frac{-y \sin x - \sin y}{x \cos y - \cos x}}$$