$$\lim_{h\to 0} \frac{3(x+h)^4-3x^4}{h}$$

$$= \lim_{h\to 0} \frac{3(\cancel{x^4}+4x^3h+6x^2h^2+4xh^3+h^4)-\cancel{3x^4}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(12x^3+\cancelto{0}{18x^2h}+\cancelto{0}{12xh^2}+\cancelto{0}{h^3})}{\cancel{h}} = 12x^3$$

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② $6x^5$ ③ $\frac{1}{3x^{2/3}}$ ④ $\frac{-3}{x^4}$

⑤ $\frac{-1}{2x^{3/2}}$ ⑥ $\frac{3}{2}x^{1/2}$ ⑦ $\frac{-3}{5x^{8/5}}$

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*7 $f(x) = \frac{1}{x^{3/5}} = x^{-3/5}$ $\quad -\frac{3}{5}-1 = \frac{-3}{5}-\frac{5}{5}$

$$f'(x) = \underbrace{-\frac{3}{5}x}\,^{-8/5} = \frac{-3}{5x^{8/5}}$$

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⑧ $18x$ ⑨ $\frac{-4}{x^2}$ ⑩ $\frac{-20}{3x^{11}}$

⑪ $\frac{-1}{12x^{4/3}}$ ⑫ $\frac{1}{x^{3/4}}$ ⑬ $\frac{18}{5x^{2/5}}$

#11. $f(x) = \frac{1}{4\sqrt[3]{x}} = \frac{1}{4}x^{-1/3}$ $-\frac{1}{3} - 1 = -\frac{1}{3} - \frac{3}{3}$

$$f'(x) = -\frac{1}{12}x^{-4/3} = \frac{-1}{12x^{4/3}}$$

<u>#20</u> $f(x) = \frac{4x^3 - x^2 + 6}{3x^2} = \frac{4x^3}{3x^2} - \frac{\cancel{x^2}}{3\cancel{x^2}} + \frac{6}{3x^2}$

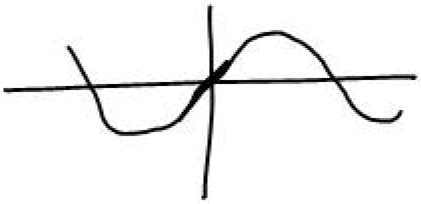
$$= \underbrace{\frac{4}{3}x - \frac{1}{3}} + \underline{2x^{-2}}$$

$$f'(x) = \frac{4}{3} - 4x^{-3} = \frac{4}{3} - \frac{4}{x^3}$$

<u>#16</u> 0

Special Limits to Know & Love

(1) $$\lim_{h\to 0}\frac{\sin h}{h} = 1$$



(2) $$\lim_{h\to 0}\frac{\cos h - 1}{h} = 0$$

(3) $$\lim_{h\to 0}\frac{e^h - 1}{h} = 1$$

Find the derivative of $f(x) = \cos x$.

$$f'(x) = \lim_{h\to 0}\frac{\cos(x+h) - \underline{\cos x}}{h}$$

$$= \lim_{h\to 0}\frac{\underline{\cos x\cosh} - \sin x\sin h - \underline{\cos x}}{h}$$

$$= \lim_{h\to 0}\left[\cos x\cdot\overset{\nearrow 0}{\frac{\cosh - 1}{h}} - \sin x\cdot\overset{\nearrow 1}{\frac{\sin h}{h}}\right]$$

$$= -\sin x$$

Find $f'(x)$ for $f(x) = e^x$

$$f'(x) = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \to 0} \frac{e^x \cdot e^h - e^x}{h}$$

$$= \lim_{h \to 0} e^x \overset{\nearrow 1}{\boxed{\frac{(e^h - 1)}{h}}} = e^x$$

The Product Rule

If $f(x) = u(x) \cdot v(x)$

then $f'(x) = \underline{u(x)} \cdot \underline{v'(x)} + \underline{v(x) \cdot u'(x)}$

#2f $f(x) = \sin x \cdot \cos x$

$$f'(x) == \underline{\sin x \cdot \sin x} + \underline{\cos x \cdot \cos x}$$

$$= \cos^2 x - \sin^2 x$$

$$= \cos 2x$$

Prove the Product Rule

$$f(x) = u(x) \cdot v(x)$$

$$f'(x) = \lim_{h \to 0} \frac{u(x+h)\,v(x+h) - u(x+h)\,v(x) + u(x+h)\,v(x) - u(x)\,v(x}{h}$$

$$= \lim_{h \to 0} \left[u(x+h) \cdot \frac{v(x+h) - v(x)}{h} + v(x) \cdot \frac{u(x+h) - u(x)}{h} \right]$$

$$= u(x) \cdot v'(x) + v(x) \cdot u'(x)$$

The Quotient Rule

If $f(x) = \dfrac{u(x)}{v(x)}$

then $f'(x) = \dfrac{v(x)\,u'(x) - u(x)\,v'(x)}{[v(x)]^2}$

Ex. #<u>3d</u> $f(x) = \frac{3x+4}{5x-1} \leftarrow$

$$f'(x) = \frac{(5x-1)(3) - (3x+4)(5)}{(5x-1)^2} \leftarrow \text{NEVER, EVER square out}$$

$$= \frac{\cancel{15x} - 3 - \cancel{15x} - 20}{(5x-1)^2} = \frac{-23}{(5x-1)^2}$$

#3j. $f(x) = \frac{x \cdot e^x}{x+2} \leftarrow$

$$f'(x) = \frac{(x+2)(x \cdot e^x + e^x) - (x \cdot e^x)(1)}{(x+2)^2}$$

$$= \frac{x^2e^x + \cancel{xe^x} + 2xe^x + 2e^x - \cancel{xe^x}}{(x+2)^2}$$

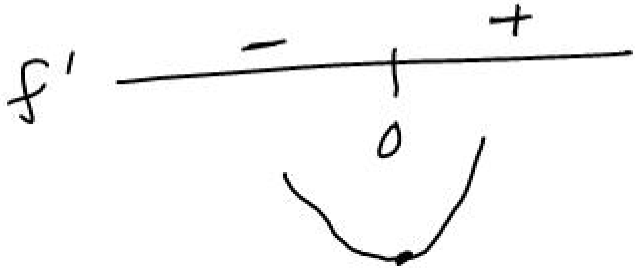
$$= \frac{e^x(x^2 + 2x + 2)}{(x+2)^2} > 0 \quad 2^2 - 4(1)(2)$$

#3e

$$f'(x) = \frac{(x^2+2)(2x) - (x^2-2)(2x)}{(x^2+2)^2}$$

$$= \frac{(2x)[(\cancel{x^2}+2) - (\cancel{x^2}-2)]}{(x^2+2)^2}$$

$$= \frac{2x(4)}{(x^2+2)^2} = \frac{8x}{(x^2+2)^2}$$



$$f''(x) = \frac{\underline{(x^2+2)^2}(8) - \underline{8x}(2)\underline{(x^2+2)^1}\underline{(2x)}}{(x^2+2)^4}$$

$$= \frac{\cancel{(x^2+2)}(8)[(x^2+2) - 4x^2]}{(x^2+2)^{\cancel{4}3}}$$

$$= \frac{8(2-3x^2)}{(x^2+2)^3} = 0 \longrightarrow x = \pm\sqrt{\frac{2}{3}}$$