Ex. $f(x) = x^3 - x$ Find $f'(x)$ and $f''(x)$

$$f'(x) = \lim_{h \to 0} \frac{[(x+h)^3 - (x+h)] - [x^3 - x]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x} - h - \cancel{x^3} + \cancel{x}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}[3x^2 + 3xh + h^2 - 1]}{\cancel{h}} = \underline{\underline{3x^2 - 1}}$$

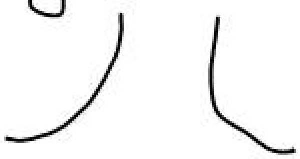
$$f''(x) = \lim_{h \to 0} \frac{[3(x+h)^2 - 1] - [3x^2 - 1]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{1} - \cancel{3x^2} + \cancel{1}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}[6x + 3h]}{\cancel{h}} = \underline{\underline{6x}}$$

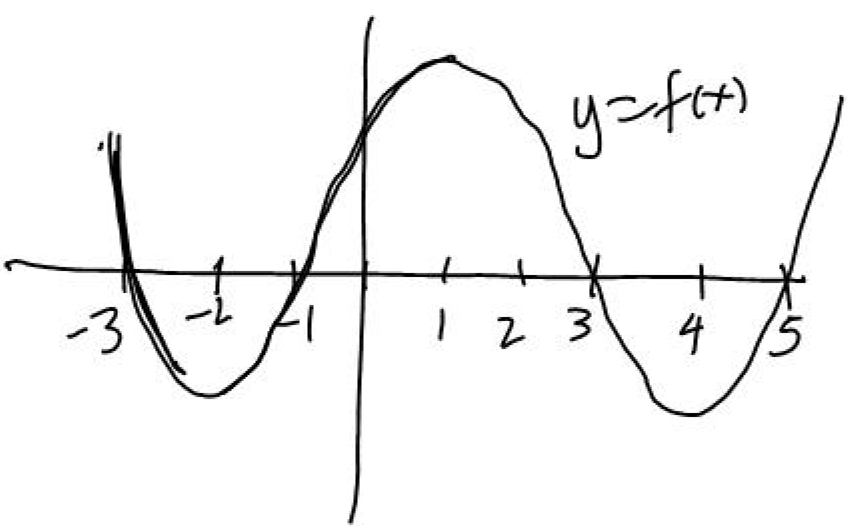
$f''(x)$ or the 2nd derivative

$g''(x) > 0$



$g''(x) < 0$

$g''(x) = 0$

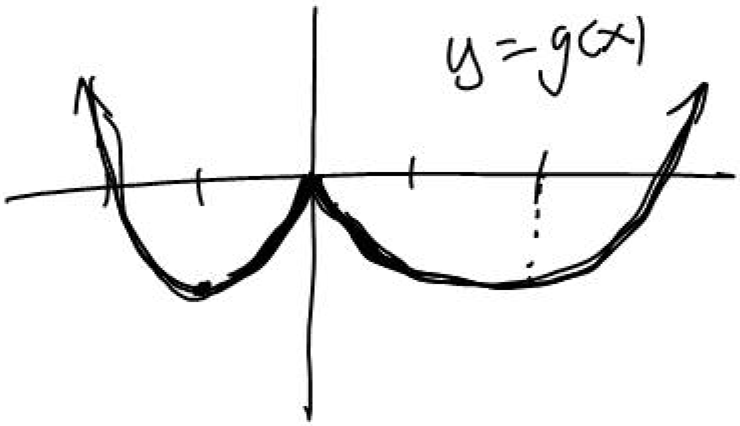


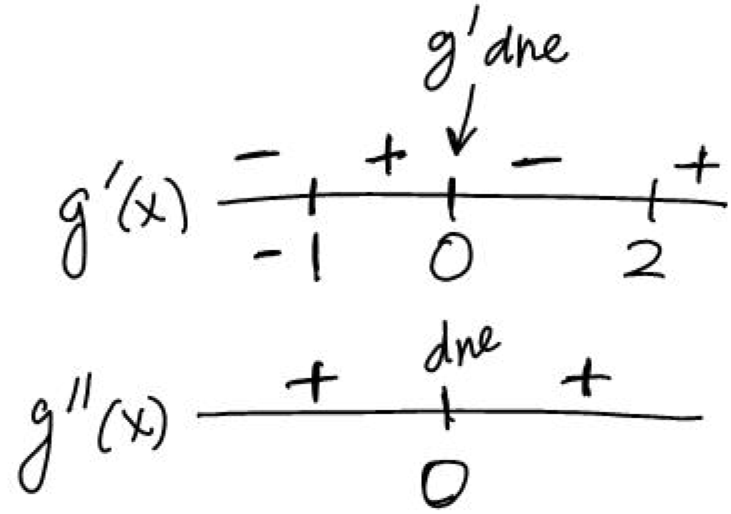
Derivative sign lines

$f'(x)$: $-$ | -2 | $+$ | 1 | $-$ | 4 | $+$

$f''(x)$: $+$ | -1 | $-$ | 3 | $+$

Two kinds of maxima:





#4 (a) $[-2, 4]$ because $f'(x) > 0$

(b) $x = -2$ because f' changes from − to +

(c) $x = 4$ because f' changes from + to −

(d) $(-\infty, 1]$ because f' is increasing

(e) $x = 1$

Wed – Quiz on calculating $f'(x)$

by $\lim_{h \to 0}$ or $\lim_{x \to a}$

optional practice

① Find $f'(x)$.

(a) $f(x) = 1 - 2x - 3x^2$

(b) $f(x) = \dfrac{2x+1}{x-4}$

(c) $f(x) = \sqrt{5-2x}$

② Find $f'(a)$.

(a) $f(x) = 2x^3 + x^2$ at $x = 2$

(b) $f(x) = \sqrt{3x+1}$ at $x = 5$

HW Set Three #2a, 3, 5, 6
