#2 $\lim_{x\to -4} \dfrac{2x^2+5x-12}{3x^2+11x-4}$

$$= \lim_{x\to -4} \frac{(2x-3)\cancel{(x+4)}}{(3x-1)\cancel{(x+4)}} = \frac{-11}{-13}$$

$$= \frac{11}{13}$$

#8 $\lim_{x\to 0} \dfrac{\frac{1}{x+2}-\frac{1}{2}}{x}$

$$= \lim_{x\to 0} \frac{\frac{2}{2(x+2)} - \frac{1(x+2)}{2(x+2)}}{\frac{x}{1}} = \lim_{x\to 0} \frac{\cancel{x}}{2(x+2)} \cdot \frac{1}{\cancel{x}}$$

$$= \frac{1}{4}$$

$$\frac{f(3) = 2(3)^2 - 3 + 4 = 19}{}$$

$f(x) = 2x^2 - x + 4 \qquad f'(3)$

$$\lim_{x\to 3} \frac{(2x^2-x+4)-(19)}{x-3} = \lim_{x\to 3} \frac{\overset{(2x+5)\cancel{(x-3)}}{2x^2-x-15}}{\cancel{x-3}}$$

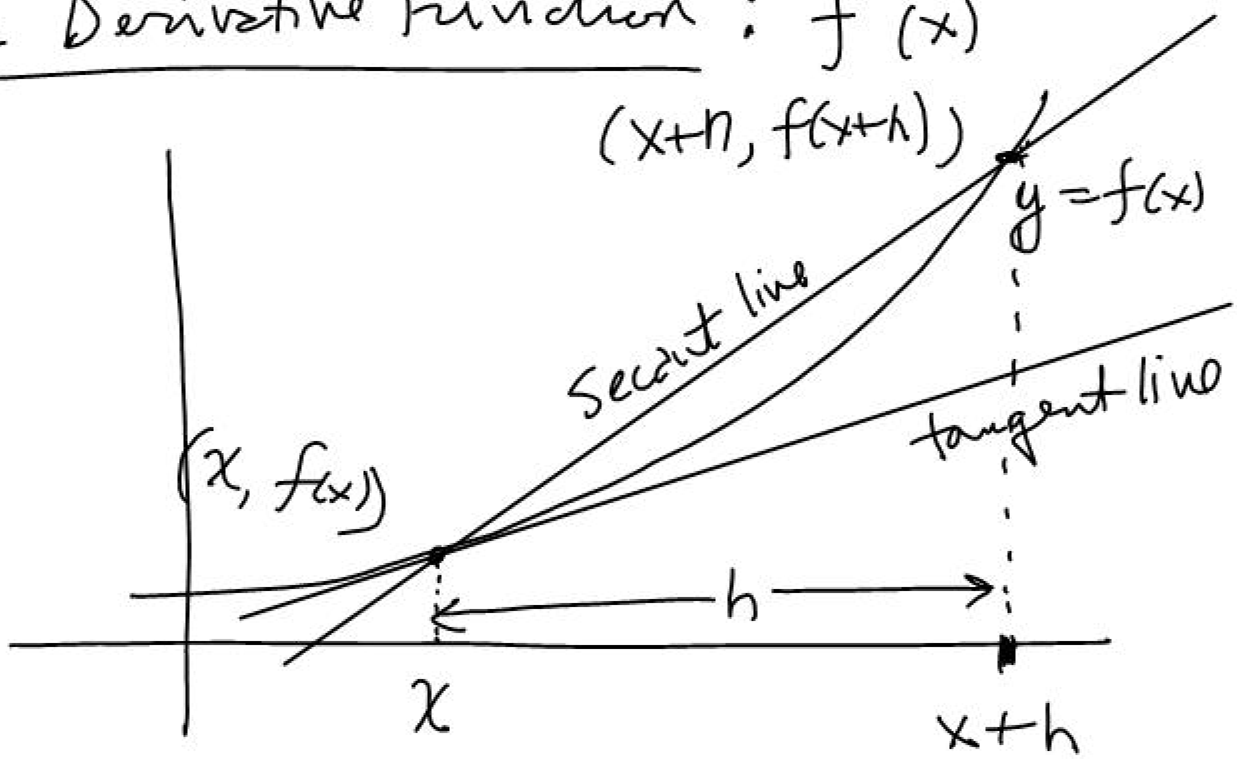
$$= 11$$

Derivatives

The derivative at a point:

$$\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

The Derivative Function: $f'(x)$



secant line slope: $\dfrac{f(x+h) - f(x)}{(x+h) - x}$

tangent line slope: $\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$

The derivative function

Ex. Find the derivative function for

$$f(x) = 2x^2 - x + 4$$

$$f'(x) = \lim_{h \to 0} \frac{\left[2\overset{x^2+2xh+h^2}{\cancel{(x+h)^2}} - (x+h) + 4\right] - \left[2x^2 - x + 4\right]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{- x} - h \cancel{+ 4} \cancel{- 2x^2} \cancel{+ x} \cancel{- 4}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left[4x + 2h - 1\right]}{\cancel{h}} = 4x - 1$$

HW

$f(3) = 11$

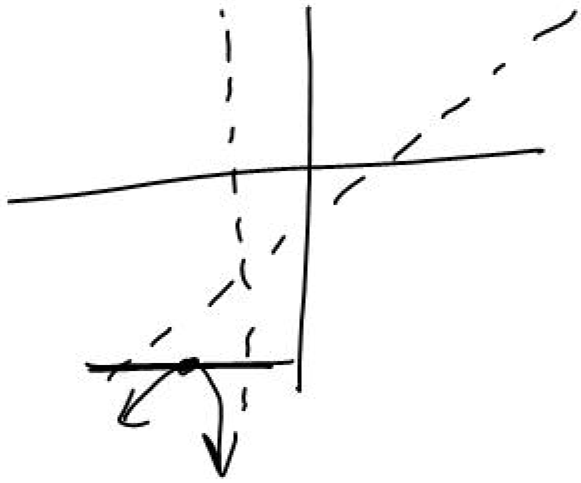
$$f'(3) = 4(3) - 1 = 11$$

$$f'(0) = -1$$

$$f'(x) = 0$$

$$4x - 1 = 0$$

$x = 1/4 \leftarrow$ x-coord. of the vertex



Ex. Find $f'(x)$ for $f(x) = \sqrt{2x+1}$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} \cdot \frac{\sqrt{2(x+h)+1} + \sqrt{2x+1}}{\sqrt{2(x+h)+1} + \sqrt{2x+1}}$$

$$= \lim_{h \to 0} \frac{2(x+h)+1 - (2x+1)}{h\left[\sqrt{2(x+h)+1} + \sqrt{2x+1}\right]}$$

$$= \lim_{h \to 0} \frac{\cancel{2x} + 2\cancel{h} + \cancel{1} - \cancel{2x} - \cancel{1}}{\cancel{h}\left[\sqrt{2(x+h)+1} + \sqrt{2x+1}\right]} = \frac{2}{2\sqrt{2x+1}}$$

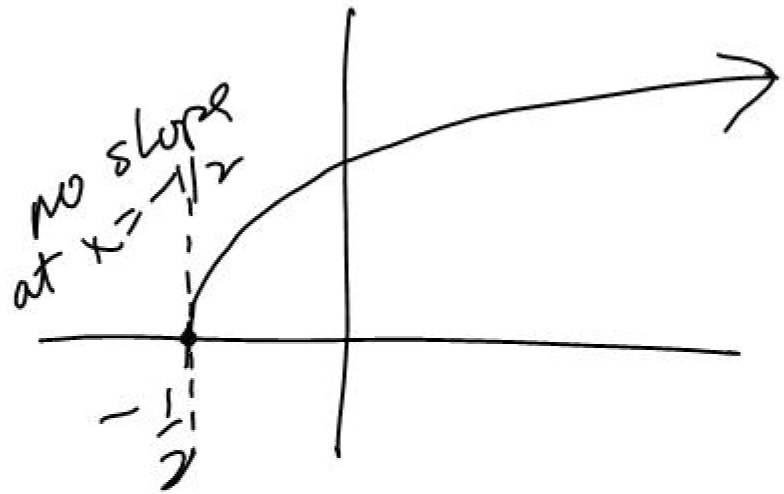
$$f'(x) = \frac{1}{\sqrt{2x+1}}$$

$$\sqrt{2x+1} = 0$$

$$2x+1 = 0$$

$$2x = -1$$

$$x = -1/2 \leftarrow \text{x-intercept}$$



Ex. find $f'(x)$ for $f(x) = \dfrac{x}{x+2}$

$$f'(x) = \lim_{h \to 0} \frac{\dfrac{x+h}{x+h+2} - \dfrac{x}{x+2}}{h}$$

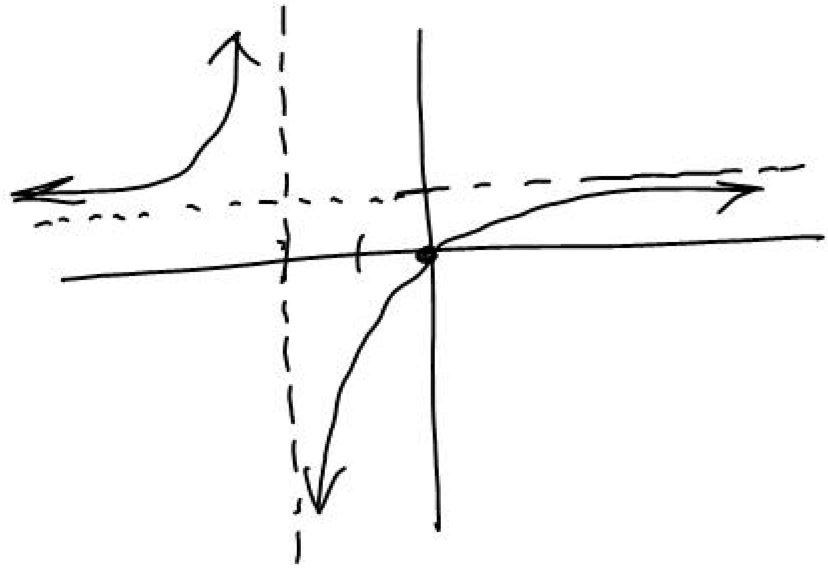
$$= \lim_{h \to 0} \frac{\dfrac{(x+h)(x+2)}{(x+h+2)(x+2)} - \dfrac{x(x+h+2)}{(x+2)(x+h+2)}}{h}$$

$$= \lim_{h \to 0} \frac{\dfrac{(x+h)(x+2)}{(x+h+2)(x+2)} - \dfrac{x(x+h+2)}{(x+2)(x+h+2)}}{h}$$

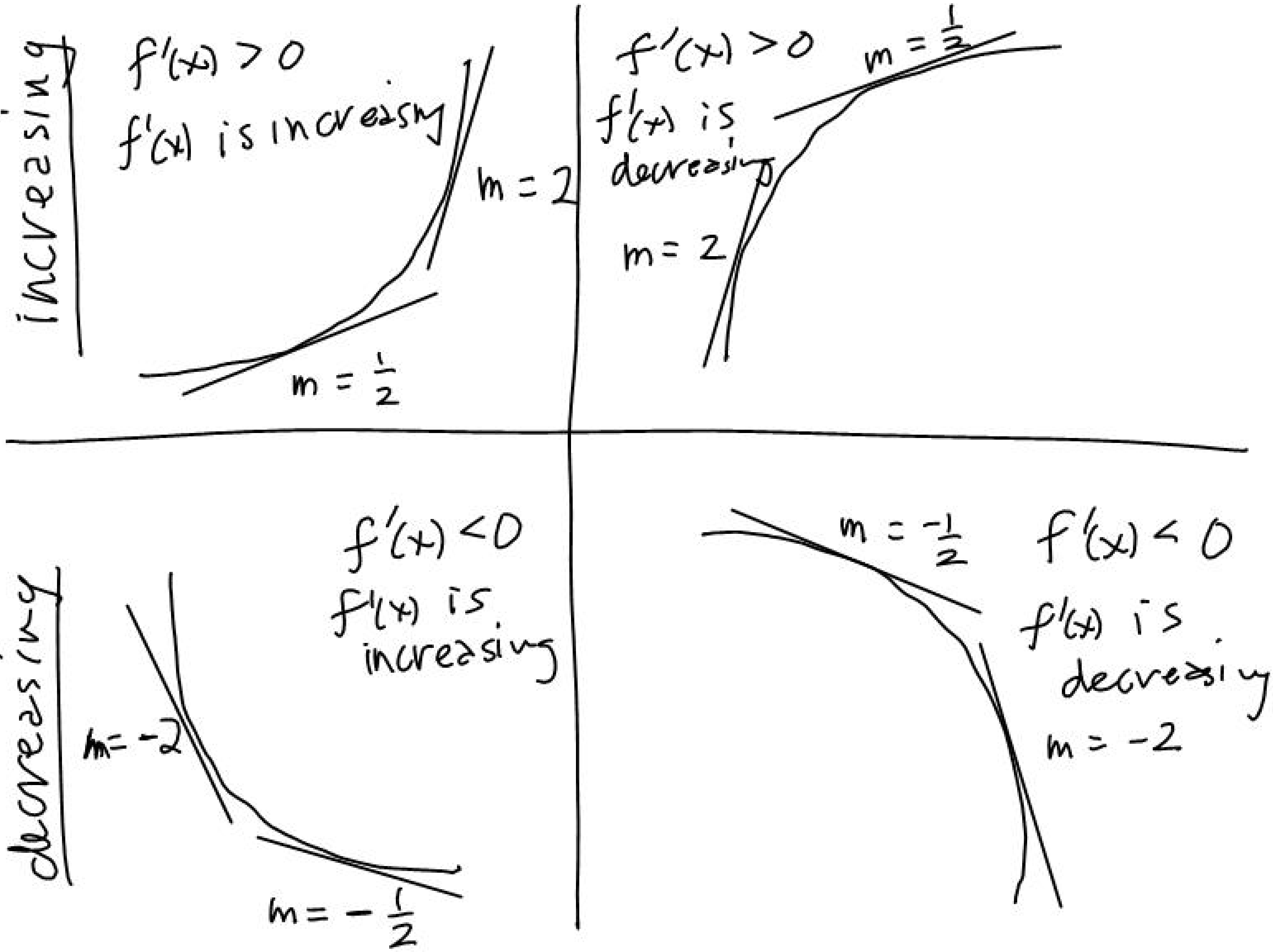
$$= \lim_{h \to 0} \frac{(\cancel{x^2} + \cancel{2x} + \cancel{xh} + 2\cancel{h}) - (\cancel{x^2} + \cancel{xh} + \cancel{2x})}{(x+h+2)(x+2)} \cdot \frac{1}{\cancel{h}}$$

$$= \frac{2}{(x+2)^2}$$

$$y = \frac{x}{x+2}$$



The four non-linear shapes

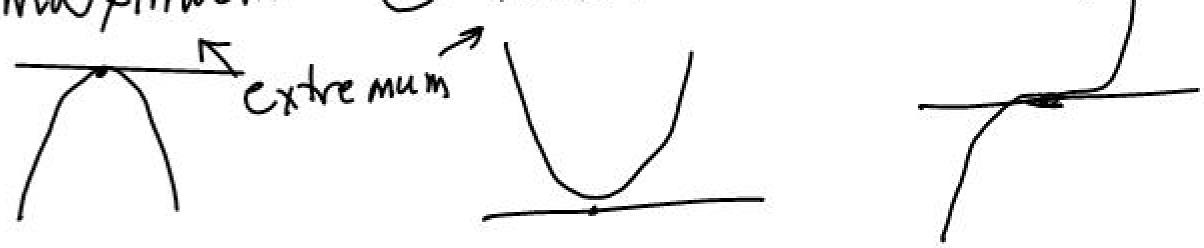


concave up

concave down

What happens when $f'(x) = 0$

① maximum ② minimum ③ flex point



HW.

(1) Find $f'(x)$, the derivative function

(a) $f(x) = 3x^2 - 5x + 2$

(b) $f(x) = 1 - x^3$

(c) $f(x) = \sqrt{4x-1}$

(d) $f(x) = \dfrac{x+2}{2x-1}$

(2) If $f(x) = x^4 - x^2 + 1$, find $f'(2)$.

(Do <u>not</u> find the deriv. function)