

2b

$$\begin{array}{l} \left[\begin{array}{cccc} 1 & -5 & 3 & -1 \\ 3 & -1 & 2 & 4 \\ 2 & 1 & -1 & 2 \\ 3 & 15 & -9 & 3 \\ 2 & 10 & -6 & 2 \end{array} \right] \xrightarrow{\substack{-3R_1 \\ -2R_1}} \left[\begin{array}{cccc} 1 & -5 & 3 & -1 \\ 0 & 14 & -7 & 7 \\ 0 & 11 & -7 & 4 \end{array} \right] \end{array}$$

$$\rightarrow \left[\begin{array}{cccc} 1 & -5 & 3 & -1 \\ 0 & 2 & -1 & 1 \\ 0 & 11 & -7 & 4 \end{array} \right] \xrightarrow{\substack{x+2 \\ x-2}} \left[\begin{array}{cccc} 1 & -5 & 3 & -1 \\ 0 & 22 & -11 & 11 \\ 0 & -22 & 14 & -8 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{cccc} 1 & -5 & 3 & -1 \\ 0 & 2 & -1 & 1 \\ 0 & 0 & 3 & 3 \end{array} \right] \rightarrow \begin{array}{l} z = 1 \\ 2y - z = 1 \\ y = 1 \end{array}$$

$$x - 5 + 3 = -1$$

$$x = 1$$

#7b

$$3x^2 - 2x + 1 + \frac{2x+3}{x^2+2x+3}$$

$$\begin{array}{r} \underline{x^2+2x+3} \overline{) 3x^4 + 4x^3 + 6x^2 - 2x + 6} \\ \underline{-3x^4 - 6x^3 - 9x^2} \\ -2x^3 - 3x^2 - 2x \\ \underline{+2x^3 + 4x^2 + 6x} \\ x^2 + 4x + 6 \\ \underline{-x^2 - 2x - 3} \\ 2x + 3 \end{array}$$

$$6 \overline{) 2 \frac{5}{6}}$$

$$6(2 \frac{5}{6}) = 12 + 5 = 17$$

degree 1 2x + 3

$$\begin{array}{r} \underline{x^2+x+1} \overline{) x^4 - x^3 + x - 1 + \frac{x}{x^2+x+1}} \\ \underline{-x^6 - x^5 - x^4} \\ -x^5 - x^4 \\ \underline{+x^5 + x^4 + x^3} \\ x^3 + x \\ \underline{-x^3 - x^2 - x} \\ -x^2 - 1 \end{array}$$

$$5 \overline{) 14 \frac{4}{5}} \begin{array}{r} 74 \\ \underline{5} \\ 24 \\ \underline{20} \\ 4 \end{array}$$

$$\begin{array}{r} \cancel{x^2} - x \\ \underline{+x^2 + x + 1} \\ x \end{array}$$

$$\text{Ex. } \begin{cases} 3x - 2y + z = -5 \\ 4x + 6y - 2z = 24 \\ 6x + 9y - 3z = 36 \end{cases}$$

$$D = \det \begin{pmatrix} 3 & -2 & 1 \\ 4 & 6 & -2 \\ 6 & 9 & -3 \end{pmatrix} = 0$$

Try Gaussian Elimination

$$\begin{array}{l} \begin{array}{l} -4 \\ 3 \end{array} \begin{bmatrix} 3 & -2 & 1 & -5 \\ \textcircled{4} & 6 & -2 & 24 \\ 6 & 9 & -3 & 36 \end{bmatrix} \rightarrow \begin{bmatrix} -12 & 8 & -4 & 20 \\ 12 & 18 & -6 & 72 \\ 6 & 9 & -3 & 36 \\ -2R_1 & -6 & 4 & -2 & 10 \end{bmatrix} \end{array}$$

$$\rightarrow \begin{bmatrix} 3 & -2 & 1 & -5 \\ \textcircled{0} & 26 & -10 & 92 \\ \textcircled{0} & 13 & -5 & 46 \\ -13 & 5 & -46 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & -2 & 1 & -5 \\ \textcircled{0} & 13 & -5 & 46 \\ \textcircled{0} & \textcircled{0} & \textcircled{0} & \textcircled{0} \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 & 1 & -5 \\ 0 & 13 & -5 & 46 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow 13y - 5z = 46$$

$$z = \frac{46 - 13y}{-5}$$

$$3x - 2y + z = -5$$

$$3x + 2y + \left(\frac{13}{5}y - \frac{46}{5}\right) = -5$$

$$z = \frac{13y - 46}{5}$$

$$-\frac{25}{5} + \frac{46}{5}$$

$$3x + \frac{23}{5}y = \frac{21}{5}$$

$$3x = \frac{21}{5} - \frac{23}{5}y$$

$$x = \frac{21}{15} - \frac{23}{15}y$$

Solution: $\left(\frac{21}{15} - \frac{23}{15}y, y, \frac{13}{5}y - \frac{46}{5}\right)$

There

$$\text{Ex. } \begin{cases} 3x + y + z = 1 \\ 2x - y - 2z = 3 \\ x + 2y + kz = 5 \end{cases}$$

(a) Is there a value of k for which the system has a unique solution?

$$D = \begin{pmatrix} 3 & 1 & 1 & 3 & 1 \\ 2 & -1 & -2 & 2 & -1 \\ 1 & 2 & k & 1 & 2 \end{pmatrix} \begin{matrix} (-3k-2+4) - (-1-12+2k) \\ \\ \end{matrix} = -5k+15$$

$$\begin{aligned} -5k+15 &\neq 0 \\ k &\neq 3 \end{aligned}$$

$$\boxed{\{k \mid k \neq 3\}}$$

$k=3 \Rightarrow$ either no solution or infinitely many

of range

$$(-\infty, -0.25] \cup (0, \infty)$$

$$\#2 \quad f(g(h(3))) = f(g(4)) = f(2) = \underline{3}$$

$$h^{-1}(g^{-1}(f^{-1}(3))) = 3$$

$$\frac{f}{(2, 3)}$$

$$\frac{f^{-1}}{(3, 2)}$$

$$f^{-1}(3) = 2$$

$$\boxed{\#3} \quad (f \circ f)(x) = f(f(x))$$

$$= f\left(\frac{1}{x-1}\right) = \frac{1}{\frac{1}{x-1} - 1} = \frac{1}{\frac{1}{x-1} - \frac{x-1}{x-1}}$$

$$= \frac{1}{\frac{2-x}{x-1}} = \frac{x-1}{2-x} \quad \frac{3-1}{2-3} = \frac{2}{-1}$$

check
 ~~$f(f(2))$~~
 ~~$f\left(\frac{1}{2-1}\right) = f(1)$~~
 $f(f(3))$
 $= f\left(\frac{1}{2}\right)$
 $= \frac{1}{\frac{1}{2} - 1} = -2$

inverse: $x = \frac{y-1}{2-y}$

$$2x - xy = y - 1$$

$$+ xy + y = +2x + 1$$

$$y(x+1) = 2x+1$$

$$(f \circ f)^{-1}(x) = \frac{2x+1}{x+1}$$

HW Review book
 #4, 5a-d, b,
 $\begin{matrix} \circ & \circ \\ \circ & \circ \\ \circ & \circ \end{matrix}$ #5
 Chapter 3R #2c Gauss