$$f(10) = 10000a + 1000b + 100c + 10d + \cancel{e}$$

$$\begin{array}{cccc} \uparrow & \uparrow & \uparrow & \uparrow \\ 3 & 2 & 5 & 8 \end{array}$$

$\boxed{\#1}$ $f(2x-1) = 16x^4 - 32x + 12x^2$

$$f = a(2x-1)^4 + b(2x-1)^3 + c(2x-1)^2 + d(2x-1) + e$$

$$= 1\cancel{a}\,(16x^4 - 32x^3 + 24x^2 - 8x \boxed{+1})$$

$$\cancel{+ \overset{0}{b}\,(8x^3 - 12x^2 + 6x - 1)}$$

$$\boxed{x^4 - 3x^2 - 2x}$$

$$\overset{-3}{\cancel{+c}}\,(4x^2 - 4x \boxed{+1})$$

$$16x^4 - 32x^3 + 12x^2 + 0x + 0$$

$$\overset{-2}{\cancel{+d}}\,(2x \boxed{+1})$$

$$\boxed{+e}$$

$$8b - 32 = -32$$

$$24 + 4c = 12$$

$$4c = -12$$

$$c = -3$$

$$1 + (-3) + 2 + e = 0$$

$$e = 0$$

$$-8 + 12 + 2d = 0$$

$$2d = -4 \rightarrow d = -2$$

Cramer's Rule

To get the determinant of a 2×2 matrix:

$$2 - -20$$

$$\det\begin{bmatrix} 2 & -5 \\ 4 & 1 \end{bmatrix} = 22$$

For a 3 × 3 determinant:

$$(0+16-6)-(-2+8+0)$$

$$\det\begin{pmatrix} 1 & 2 & -1 \\ 3 & 1 & 4 \\ 2 & 2 & 0 \end{pmatrix} \begin{matrix} 1 & 2 \\ 3 & 1 \\ 2 & 2 \end{matrix} = 10 - 6 = 4$$

$$\begin{cases} ax + by + cz = d \\ ex + fy + gz = h \\ ix + jy + kz = l \end{cases}$$

$$x =$$

Ex Solve $\begin{cases} 2x + y = 7 \\ 3x - 2y = -7 \end{cases}$

$2(1)+5=7$

$3(1)-2(5)=-7$

Cramer's Rule

$$D = \det\begin{pmatrix} 2 & 1 \\ 3 & -2 \end{pmatrix} = -7$$

$-14 - -7$

$$D_x = \det\begin{pmatrix} 7 & 1 \\ -7 & -2 \end{pmatrix} = -7$$

$-14 - 21$

$$D_y = \det\begin{pmatrix} 2 & 7 \\ 3 & -7 \end{pmatrix} = -35$$

$$x = \frac{D_x}{D} = \frac{-7}{-7} = 1$$

$$y = \frac{D_y}{D} = \frac{-35}{-7} = 5$$

3x3 system

EX
$$\begin{cases} x + 2y - z = -3 \\ 2x + y + 3z = 12 \\ 3x - 4y - 2z = 4 \end{cases}$$

Cramer's Rule

$$(-2 + 18 + 8) - (-3 - 12 - 8)$$

$$D = \det \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 3 \\ 3 & -4 & -2 \end{pmatrix} \begin{matrix} 1 & 2 \\ 2 & 1 \\ 3 & -4 \end{matrix} = 24 - {}^{-}23 = 47$$

$$(6 + 24 + 48) - (-4 + 36 - 48)$$

$$D_x = \det \begin{pmatrix} -3 & 2 & -1 \\ 12 & 1 & 3 \\ 4 & -4 & -2 \end{pmatrix} \begin{matrix} -3 & 2 \\ 12 & 1 \\ 4 & -4 \end{matrix} = 78 - {}^{-}16 = 94$$

$$\boxed{x = \frac{94}{47} = 2}$$

$$(-24-27-8)-(-36+12+12)$$

$$D_y = \det \begin{pmatrix} 1 & -3 & -1 \\ 2 & 12 & 3 \\ 3 & 4 & -2 \end{pmatrix} \begin{matrix} 1 & -3 \\ 2 & 12 \\ 3 & 4 \end{matrix} = -47$$

$$\boxed{y = \frac{-47}{47} = -1}$$

$$2 + 2(-1) - z = -3$$

$$\boxed{z = 3}$$

Solve with Gaussian Elimination

$$\begin{cases} x + 2y - z = -3 \\ 2x + y + 3z = 12 \\ 3x - 4y - 2z = 4 \end{cases}$$

Start with the augmented matrix – get zeros in the bottom left-hand corner

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 1 & 2 & -1 & -3 \\ 2 & 1 & 3 & 12 \\ 3 & -4 & -2 & 4 \end{bmatrix}$$

$$\begin{array}{ccccc} -2R_1 & -2 & -4 & +2 & +6 \\ -3R_1 & -3 & -6 & +3 & +9 \end{array}$$

$$\begin{array}{c} R_1 \\ 10R_2 \\ -3R_3 \end{array} \begin{bmatrix} 1 & 2 & -1 & -3 \\ 0 & -3 & 5 & 18 \\ 0 & -10 & 1 & 13 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -1 & -3 \\ 0 & -30 & 50 & 180 \\ 0 & 30 & -3 & -39 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 2 & -1 & -3 \\ 0 & -3 & 5 & 18 \\ 0 & 0 & 47 & 141 \end{bmatrix}$$

R3: $47z = 141 \rightarrow z = \frac{141}{47} = 3$

R2: $-3y + 5(3) = 18$

$-3y = 3$

$y = -1$

R_1: $1x + 2(-1) - 1(3) = -3$

$x = 2$

$\boxed{3R}$ #1 Cramer's Rule

#2 a Cramer's Rule

b Gaussian Elimination

$\boxed{3\ \mathrm{I}}$ #1a

$$
\begin{array}{r|l}
 & x^3 + 3x^2 + 2x - 1 \\
\hline
x+2 & \cancel{x^4} + 5x^3 + 8x^2 + 3x - 2 \\
 & -\cancel{x^4} - 2x^3 \\
\hline
 & \cancel{3x^3} + 8x^2 \\
 & -\cancel{3x^3} - 6x^2 \\
\hline
 & \cancel{2x^2} + 3x \\
 & -\cancel{2x^2} - 4x \\
\hline
 & -x - 2 \\
 & +x + 2 \\
\hline
 & 0
\end{array}
$$

$$
\begin{array}{r|l}
5 & 97 \\
 & -5 \\
\hline
 & 47
\end{array}
$$

$$
\begin{array}{r|rrrrr}
-2 & 1 & 5 & 8 & 3 & -2 \\
 & & -2 & -6 & -4 & 2 \\
\hline
 & 1 & 3 & 2 & -1 & \boxed{0}
\end{array}
$$

$$\boxed{x^2 + 5x + 9 + \frac{8x+7}{x^2-1}}$$

$x^2 - 1$

quadratic
No
sythetic
shortcut

$$\cancel{x^4} + 5x^3 + 8x^2 + 3x - 2$$

$$-\cancel{x^4} \qquad + x^2$$

$$\cancel{5x^3} + 9x^2 + 3x$$

$$-\cancel{5x^3} \qquad + 5x$$

$$\cancel{9x^2} + 8x - 2$$

$$-\cancel{9x^2} \qquad + 9$$

$$8x + 7$$

$\boxed{3\text{I}}$ ~~#1 b, c~~ #2 b, c