Polynomials

(If $\frac{p}{q}$ is a root, then p divides evenly into the constant and q divides evenly into the leading coefficient)

Ex, Find the roots of

$$f(x) = x^4 - 15x^3 + 81x^2 - 189x + 162 = 0$$

Is $x-1$ a factor?

1	1	−15	81	−189	162
		1	−14	67	−122
	1	−14	67	−122	40 $= f(1)$

2	1	−15	81	−189	162
		2	−26	110	−158
	1	−13	55	−79	4 $= f(2)$

3	1	−15	81	−189	162
		3	−36	135	−162
	1	−12	45	−54	0 $= f(3)$

$$(x-3)(x^3 - 12x^2 + 45x - 54) = 0$$

3	1	−12	45	−54
		3	−27	54
	1	−9	18	0 = f(3)

$$(x-3)^2(x^2-9x+18)=0$$

$$(x-3)^2(x-3)(x-6)=0$$

$$(x-3)^3(x-6)=0$$

roots:

$r_1 = 3$

$r_2 = 3$

$r_3 = 3$

$r_4 = 6$

opp same opp same

$$x^4 - 15x^3 + 81x^2 - 189x + 162$$

$r_1 r_2 r_3 r_4 = 162$

$r_1 + r_2 + r_3 + r_4 = 15$

$r_1r_2 + r_1r_3 + r_1r_4 + r_2r_3 + r_2r_4 + r_3r_4 =$

$9 + 9 + 18 + 9 + 18 + 18 = 81$

$r_1r_2r_3 + r_1r_2r_4 + r_1r_3r_4 + r_2r_3r_4$

$27 + 54 + 54 + 54 = 189$

Ex. The roots of $\overset{\text{opp}}{x^3 - 5x^2} \overset{\text{same}}{+ 6x} \overset{\text{opp}}{- 9} = 0$

are r_1, r_2, and r_3.

evaluate: $r_1 r_2 r_3 = \underline{\ 9\ }$

$r_1 + r_2 + r_3 = \underline{\ 5\ }$

$\binom{3}{2}$ $r_1 r_2 + r_1 r_3 + r_2 r_3 = \underline{\ 6\ }$

Ex. The roots of $5x^5 - 9x^3 + x - 3 = 0$

are r_1, r_2, r_3, r_4, and r_5.

$$x^5 - \frac{9}{5}x^3 + \frac{1}{5}x - \frac{3}{5} = 0$$

$r_1 + r_2 + r_3 + r_4 + r_5 = \underline{\ 0\ }$ $(0x^4)$

$r_1 r_2 r_3 r_4 r_5 = \underline{\ +3/5\ }$

$$\frac{9}{5} = r_1r_2 + r_1r_3 + r_1r_4 + r_1r_5 + r_2r_3 + r_2r_4 + r_2r_5 + r_3r_4 + r_3r_5 + r_4r_5$$

Viète's theorem (François Viète 1540-1603)

$$\overset{\text{opp}}{4x^2 - 3x} + \overset{\text{same}}{5} = 0$$

$$r_1 + r_2 = \frac{3}{4}$$

$$r_1 \cdot r_2 = \frac{5}{4}$$

Ex- $r_1 = 3 + i$

$$r_1 + r_2 = (3+i) + (3-i) = 6$$

$$r_1 \cdot r_2 = (3+i)(3-i) = 9 - i^2 = 10$$

$$\boxed{x^2 - 6x + 10 = 0}$$

$$i^{183} = i^{180} \cdot i^3 = i \cdot i^2 = -i$$

$$\boxed{i^i = e^{-\pi/2}} \quad \sqrt{i} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \quad \boxed{e^{i\pi} + 1 = 0}$$