

The Binomial Theorem

$$\begin{aligned}(x+4)^4 &= (x+4)(x+4)(x+4)(x+4) \\ &= (x^2+8x+16)(x^2+8x+16) \\ &= \underline{x^4} + \underline{8x^3} + \underline{16x^2} + \underline{8x^3} + \underline{64x^2} + \underline{128x} + \underline{16x^2} + \underline{128x} + \underline{256} \\ &= \underline{x^4} + \underline{16x^3} + \underline{96x^2} + \underline{256x} + \underline{256}\end{aligned}$$

$$\begin{aligned}(x+4)^4 &= (x+4)(x+4)(x+4)(x+4) \\ &= x^4 + \binom{4}{1}4x^3 + \binom{6}{2}16x^2 + \binom{4}{1}64x + 256 \\ &\quad \uparrow \quad \quad \uparrow \quad \quad \uparrow \quad \quad \uparrow \quad \quad \uparrow \\ &\quad \binom{4}{4} \quad \binom{4}{3} \quad \binom{4}{2} \quad \binom{4}{1} \quad \binom{4}{0}\end{aligned}$$

Rules

$$\binom{n}{n} = \binom{n}{0} = 1$$

$$\binom{n}{r} = \binom{n}{n-r}$$

$$\binom{n}{1} = \binom{n}{n-1} = n$$

$$\boxed{\binom{n}{r} = \frac{n!}{r!(n-r)!}}$$

$$\frac{5 \cdot 4}{5!} = \frac{2}{3! \cdot 2!}$$

$$\begin{aligned} (2x - 3y)^5 &= (2x - 3y)(2x - 3y)(2x - 3y)(2x - 3y)(2x - 3y) \\ &= (2x)^5 + \binom{5}{4} (2x)^4 (-3y)^1 + \binom{5}{3} (2x)^3 (-3y)^2 \\ &\quad + \binom{5}{2} (2x)^2 (-3y)^3 + \binom{5}{1} (2x)^1 (-3y)^4 + (-3y)^5 \\ &= 32x^5 - 240x^4y + 720x^3y^2 - 1080x^2y^3 \\ &\quad + 810xy^4 - 243y^5 \end{aligned}$$

HW ① $(3x - 2y)^6$

② $\left(4x^2 - \frac{3}{x^3}\right)^{10} \longrightarrow \frac{1}{x^{10}}$