(2b) $\sum_{r=1}^{n} 2^{r-1} = 2^n - 1$

$$1 + 2 + 4 + 8 + \cdots + 2^{n-1} = 2^n - 1$$

$k+1$

<u>Step 2</u> Induction hypothesis: $1 + 2 + 4 + \cdots + 2^{k-1} = 2^k - 1$

show: $\underline{1+2+4+\cdots+2^{k-1}+2^k} = 2^{k+1}-1$

$$\underbrace{1 + 2 + 4 + \cdots + 2^{k-1}} + 2^k$$

$$= 2^k - 1 + 2^k$$

$$= 2^1 \cdot 2^k - 1$$

$$= 2^{k+1} - 1$$

2(d) $\sum_{r=1}^{n} r(r+2) = \frac{n}{6}(n+1)(2n+7)$

$2(k+1)+7$

<u>Step 2</u> Assume: $1(3) + 2(4) + 3(5) + \cdots + k(k+2) = \frac{k}{6}(k+1)(2k+7)$

Prove: $\underline{1(3) + 2(4) + \cdots + k(k+2)} + (k+1)(k+3)$

$$= \frac{k+1}{6}(k+2)(2k+9)$$

$$\frac{k}{6}(k+1)(2k+7) + (k+1)(k+3)$$

$$= (k+1)\left[\frac{k}{6}(2k+7) + \frac{6k+18}{6}\right] = \frac{(k+1)}{6}\left[\cancel{2k^2+7k+6k+18}\right]$$

$$2k^2 + 13k + 18$$

More Induction: Divisibility Proofs

Prove: $5^n - 1$ is divisible by 4.

step 1: $5^1 - 1 = 4$ ✓

Assume: $5^k - 1$ is divisible by 4

Show: $5^{k+1} - 1$ is divisible by 4

$5^k - 1 = 4z$ for $z \in \mathbb{Z} \Rightarrow 5^k = 4z + 1$

$$5^{k+1} - 1 = 5^1 \cdot 5^k - 1$$
$$= 5(4z+1) - 1$$
$$= 20z + 4$$
$$= 4 \cdot (5z + 1)$$ which is div. by 4

$4 \mid 16$ means "4 divides 16"

Prove: $3 \mid 5^n + 2 \times 11^n$

step 1: $5^1 + 2 \times 11^1 = 5 + 22 = 27$ which is div. by 3 ✓

step 2: Assume: $3 \mid 5^k + 2 \times 11^k$

Show: $3 \mid 5^{k+1} + 2 \times 11^{k+1}$

By the induction hypothesis, $5^k + 2 \times 11^k = 3z$ for $z \in \mathbb{Z}$

so, $5^k = 3z - 2 \times 11^k$

$$5^{k+1} + 2 \times 11^{k+1} = 5^1 \cdot 5^k + 2 \times 11^{k+1}$$

using the induction hypothesis

$$= 5 \cdot (3z - 2 \times 11^k) + 2 \times 11^{k+1}$$

$$\left.\begin{aligned} &= 15z - 10 \times 11^k + 2 \times 11^{k+1} \\ &= 15z - 11^k[10 - 2 \times 11] \end{aligned}\right\} \text{factor out } 11^k$$

$$= 15z - 11^k[-12]$$

$$= 15z + 12 \times 11^k$$

$$= 3[5z + 4 \times 11^k]$$ which is div. by 3

HW: 1I #1,4; quiz practice