$\boxed{1E}$ #1c $\quad 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} \cdots$ (10 terms)

geometric with $r = -\frac{1}{2}$

$$S_{10} = \frac{1\left(1 - \left(-\frac{1}{2}\right)^{10}\right)}{1 - -\frac{1}{2}} = \frac{2}{3}\left(\frac{2^{10}}{2^{10}} - \frac{1}{2^{10}}\right)$$

$$= \frac{2}{3}\left(\frac{1023}{1024}\right) = \frac{341}{512}$$

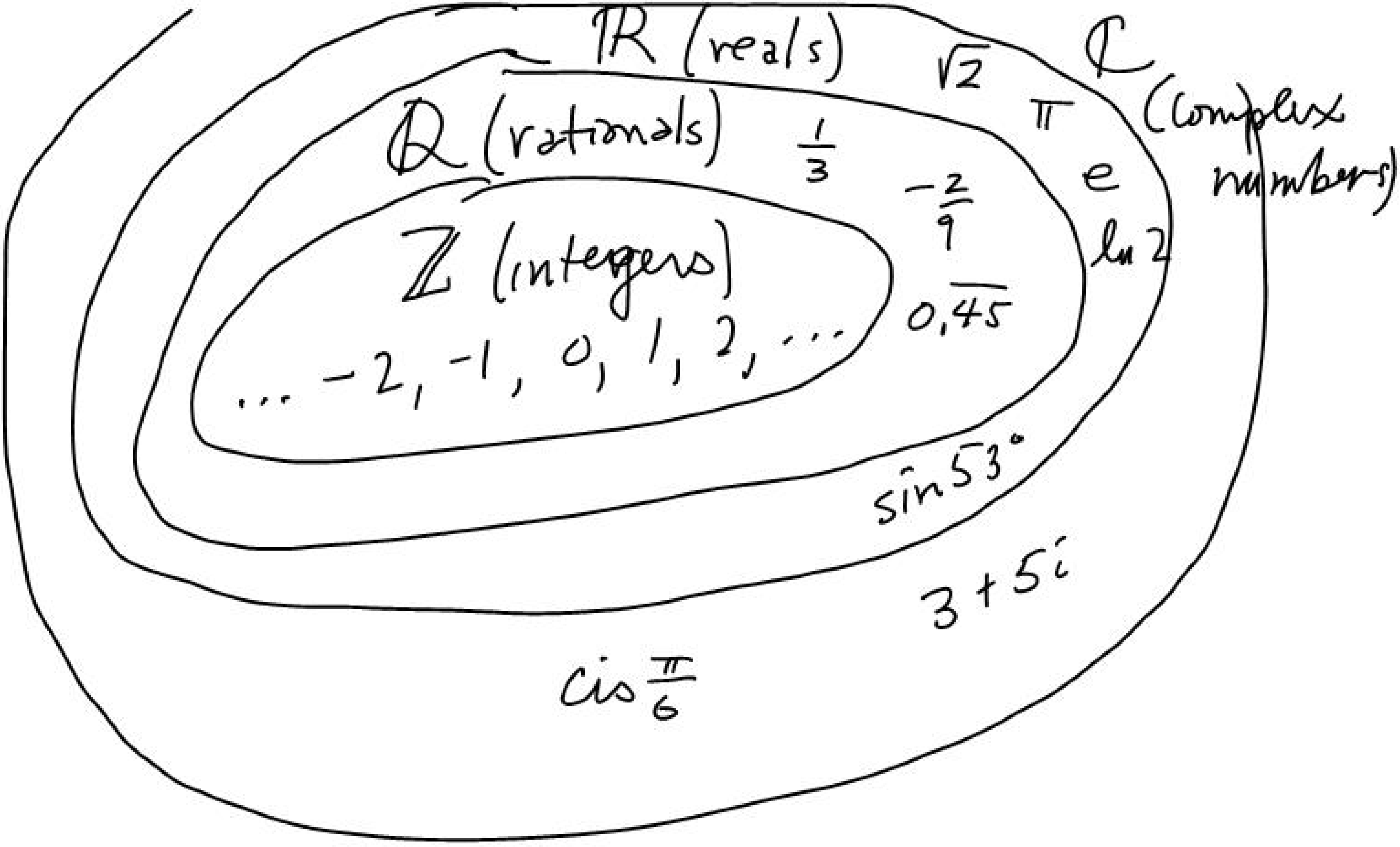
$$1 + \frac{-\frac{1}{2}\left(1 - \left(-\frac{1}{2}\right)^{9}\right)}{1 - -\frac{1}{2}}$$

$$= 1 + \frac{\cancel{2}}{3}\left(\frac{-1}{\cancel{2}}\right)\left(\frac{2^9}{2^9} + \frac{1}{2^9}\right)$$

$$= 1 + \frac{-1}{\cancel{3}}\left(\frac{\overset{171}{\cancel{513}}}{512}\right)$$

$$= \frac{512}{512} - \frac{171}{512}$$

$$= \frac{341}{512}$$



$\boxed{1E}$ #9. $S - S_n = k u_n$

$$(1-r)\left(\frac{1\cancel{u_1}}{1-r} - \frac{\cancel{u_1}(1-r^n)}{1-r}\right) = \left(k\cancel{u_1}r^{n-1}\right)(1-r)$$

$$1 - (1-r^n) = k r^{n-1}(1-r)$$

$$\frac{r^n}{r^{n-1}} = \frac{k\cancel{r^{n-1}}(1-r)}{\cancel{r^{n-1}}}$$

$$r = k(1-r)$$

$$r = k - kr$$

$$r + kr = k$$

$$r(1+k) = k$$

$$\boxed{r = \frac{k}{1+k}}$$

Mathematical Induction

(A method of writing proofs)

Ex Prove: $\underbrace{1+2+3+\cdots+n}$ $= \dfrac{n(n+1)}{2}$

Step 1: show the formula works for $n=1$.
(easy)

$$1 = \frac{1(1+1)}{2} \checkmark$$

Step 2 Assume the formula is true for $n=k$,
(hard) and show it has to be true for $n=k+1$.

Assume that $1+2+3+\cdots+k = \dfrac{k(k+1)}{2}$ (given)

Show that $1+2+3+\cdots+k+(k+1) = \dfrac{(k+1)(k+2)}{2}$ (prove)

$$\underbrace{1+2+3+\cdots+k}+(k+1)$$

$$= \frac{k(k+1)}{2} + k+1$$

$$= \frac{k(k+1)}{2} + \frac{2k+2}{2}$$

$$= \frac{k^2+k+2k+2}{2} = \frac{k^2+3k+2}{2} = \frac{(k+1)(k+2)}{2} \checkmark$$

Ex Prove: $1^3+2^3+3^3+\cdots+n^3=\frac{n^2(n+1)^2}{4}$

step 1 Show it's true for $n=1$.

$$1^3=\frac{1^2(1+1)^2}{4}$$ ✓

induction hypothesis

step 2 Assume: $1^3+2^3+3^3+\cdots+k^3=\frac{k^2(k+1)^2}{4}$

Show: $1^3+2^3+3^3+\cdots+k^3+(k+1)^3=\frac{(k+1)^2(k+2)^2}{4}$

$$1^3+2^3+3^3+\cdots+k^3+(k+1)^3$$

$$=\frac{k^2(k+1)^2}{4}+(k+1)^3$$

$$=\frac{k^2(k+1)^2+4(k+1)^3}{4}$$

$$=\frac{(k+1)^2\left[k^2+4(k+1)\right]}{4}$$

$$=\frac{(k+1)^2(k^2+4k+4)}{4}=\frac{(k+1)^2(k+2)^2}{4}$$ ✓

HW

$\boxed{1H}$

(2a) $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$

$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n}{6}(n+1)(2n+1)$

(2b) $\sum_{r=1}^{n} 2^{r-1} = 2^n - 1$

$1 + 2 + 2^2 + 2^3 + \cdots 2^{n-1} = 2^n - 1$

(2d) $\sum_{r=1}^{n} r(r+2) = \frac{n}{6}(n+1)(2n+7)$

$1(3) + 2(4) + 3(5) + \cdots + n(n+2) = \frac{n}{6}(n+1)(2n+7)$

Factorials

$$\binom{n}{r} \longrightarrow \frac{n!}{r!\,(n-r)!}$$

read "n choose r

the way to choose r things out of a set of n things.

a <u>combination</u>

Ex. How many ways can we choose 3 people out of a class of 9 ?

$$\binom{9}{3} = \frac{9!}{3!\,(9-3)!} = \frac{\overset{3}{\cancel{9}} \cdot \overset{4}{\cancel{8}} \cdot 7 \cdot \cancel{6!}}{\underset{\cancel{3} \cdot \cancel{2}}{\cancel{3!}}\ \cancel{6!}}$$

$$= 84$$

Ex. A class of 9 elects a pres., VP, and a Secretary. How many ways can this done?

<u>Permutation</u>: P^9_3 read "9 pick 3"

$$P^9_3 = \frac{9!}{(9-3)!} = (84)(3!) = 504$$

Ex. How many ways are there to line up 9 people?

$$9! = 9 \cdot 8 \cdot 7 \cdot \cdots 1$$

$$= 362,880$$