$\boxed{1A}$ # 5(a) Arithmetic : $d = 4$ , $u_1 = -1$

$$u_n = \underline{\underline{u_1 + (n-1)d}}$$

$$\sum_{n=1}^{\infty} \left[-1 + (n-1)(4)\right]$$

$$= \sum_{n=1}^{\infty} (\underline{4n - 5})$$

summand

(b) $\displaystyle\sum_{n=1}^{10} (-1)^n$

(c) Geometric : $u_1 = 6$ , $r = -2$

$$u_n = \underline{u_1 r^{n-1}}$$

$$\sum_{n=1}^{6} 6(-2)^{n-1} \quad \underline{\text{or}} \quad \sum_{n=1}^{6} (-1)^{n-1} \cdot 6 \cdot 2^{n-1}$$

<u>or</u>

$$\sum_{n=0}^{5} \underset{u_1}{6}\,(\underset{r}{-2})^n$$

$\boxed{1C}$ #2 (a) $\sum_{r=1}^{10}(5r+7) = \overset{d=5}{12+17}+22+\cdots+57$

$$= \frac{10}{2}(12+57)$$

$$= \frac{10}{2}(69) = \frac{690}{2}$$

$$= 345$$

$\boxed{1D}$ # 1b $\overset{r=\frac{1}{3}}{9,3},1,\ldots$

$$u_n = 9\cdot\left(\frac{1}{3}\right)^{n-1} = \frac{3^2}{3^{n-1}} = \frac{1}{3^{n-3}}$$

$$u_1 \cdot r^{n-1}$$

$$= 3^{3-n}$$

#6. $r = \frac{a+2}{a-4} = \frac{3a+1}{a+2}$

$r = \frac{3/\cancel{2}}{-9/\cancel{2}} = \frac{-1}{3}$ $\quad a^2+4a+4 = 3a^2-11a-4$

$$0 = 2a^2-15a-8$$

$r = \frac{10}{4} = \frac{5}{2}$ $\quad 0 = (2a+1)(a-8)$

$$a = -1/2 \text{ or } a = 8$$

# Geometric Series

$$S_n = u_1 + \cancel{u_1 r} + \cancel{u_1 r^2} + \cancel{\cdots} + \cancel{u_1 r^{n-1}}$$
$$rS_n = \cancel{u_1 r} + \cancel{u_1 r^2} + \cancel{u_1 r^3} + \cancel{\cdots} + u_1 r^n$$
subtract

$$S_n - rS_n = u_1 - u_1 r^n$$

$$S_n(1-r) = u_1(1-r^n)$$

$$\boxed{S_n = \frac{u_1(1-r^n)}{1-r}}$$

The sum of n terms of a Geometric Series

or

$$\boxed{S_n = \frac{u_1(r^n - 1)}{r-1}}$$

Ex. Find the sum of the 1st 10 terms of

8, 4, 2, 1, ...

$$S_{10} = \frac{8\left(\left(\frac{1}{2}\right)^{10} - 1\right)}{\frac{1}{2} - 1} = -16\left(\frac{1}{2^{10}} - \frac{2^{10}}{2^{10}}\right)$$

$$= +\cancel{2^4}\left(\frac{+1023}{\cancel{2^{10}}\ 2^6}\right)$$

$$= \frac{1023}{64}$$

$\boxed{\text{Ex}}$ Find the Sum : $8+4+2+1+\cdots$

$$S_\infty = \frac{8\left(1-\cancel{\left(\frac{1}{2}\right)^{\infty}}^{\,0}\right)}{1-\frac{1}{2}} \qquad \frac{1}{2^\infty}$$

$$S = \frac{8}{1-\frac{1}{2}} = \underline{\underline{16}}$$

The Sum of an infinite geometric series is

$$\boxed{S = \frac{u_1}{1-r}, \quad -1 < r < 1}$$

If $r > 1$, $8+16+32+64+\cdots$ diverges

$-1 < r < 1$ is the same thing as $\underline{|r| < 1}$

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$\boxed{\text{Ex}}$ Evaluate : $\displaystyle\sum_{n=2}^{\infty} 4\left(\frac{1}{3}\right)^n$

Solution : $\dfrac{4/9}{1-1/3} \qquad \dfrac{\cancel{4}^{\,2}/\cancel{9}_{\,3}}{\cancel{2}/\cancel{3}} = \dfrac{2}{3}$

$\boxed{\text{Ex}}$ Find the sum:

$$1 + x^2 + x^4 + x^6 + \cdots$$

The sum is $\dfrac{1}{1 - x^2}$, $\; -1 < x^2 < 1$

$\boxed{\text{IE}}$ #3. $u_1 r^2 = 2$

$$u_1 r^6 = \frac{1}{128} = 2 \cdot r^4$$

$$\frac{1}{256} = r^4$$

$$\frac{1}{2^8} = r^4$$

$$\underline{\frac{1}{4} = r}$$

$$u_1 \left(\frac{1}{4}\right)^2 = 2$$

$$u_1 \cdot \frac{1}{16} = 2$$

$$\underline{u_1 = 32}$$

$$S_6 = \frac{32\left(1 - \left(\frac{1}{4}\right)^6\right)}{1 - \frac{1}{4}} = \frac{128\left(\frac{4^6 - 1}{4^6}\right)}{3}$$

$$= \frac{2^7\left(4^6 - 1\right)}{3 \cdot 2^{12}} = \frac{4^6 - 1}{3 \cdot 2^5}$$

$\boxed{1E}$ #8. $\displaystyle\sum_{r=0}^{\infty} \frac{(x+1)^r}{3^r} = \sum_{r=0}^{\infty} \left(\frac{x+1}{3}\right)^r$

common ratio

$$= 1 + \frac{x+1}{3} + \frac{(x+1)^2}{9} + \frac{(x+1)^3}{27} + \cdots$$

Converges for $-1 < \dfrac{x+1}{3} < 1$

$$-3 < x+1 < 3$$

$$\underbrace{-4 < x < 2}_{\text{The interval of convergence for the series}}$$

Evaluate $\displaystyle\sum_{r=0}^{\infty} \left(\frac{-1.5+1}{3}\right)^r = \sum_{r=0}^{\infty} \left(\frac{-1}{6}\right)^r$

$$= \frac{1}{1 - -1/6} = \frac{6}{7}$$

## HW

① Evaluate

(a) $\sum_{n=0}^{\infty} 2\left(-\frac{1}{5}\right)^n$ (b) $\frac{2}{3} - \frac{2}{9} + \frac{2}{27} - \frac{2}{81} \cdots$

② Find the sum. State restrictions on $x$.

(a) $\sum_{n=0}^{\infty} \frac{x^n}{2^n}$ (b) $1 - (x-2) + (x-2)^2 - (x-2)^3 \cdots$

$\boxed{1E}$ #1, 2, 9

$\boxed{1F}$ # 1, 2

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$1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2 =$

$1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$