Sequences & Series (chapter 1)

1 2 3 4 5 6 7 ← term number

Sequence: $1, 1, 2, 3, 5, 8, 13, \ldots$ (n)

In this seq., $u_4 = 3$

Ex: $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots$

$u_4 = \frac{1}{27}$

$u_n = \frac{1}{3} u_{n-1}$ (recursive formula)

$u_n = \frac{1}{3^{n-1}} = \left(\frac{1}{3}\right)^{n-1}$

Series: $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$

$u_3 = \frac{1}{8}$

sum $\longrightarrow$ $S_3 = \frac{7}{8}$ (3^{rd} partial sum)

$S_5 = \frac{31}{32}$ (5^{th} partial sum)

$$S_n = \frac{2^n - 1}{2^n} = 1 - \frac{1}{2^n}$$

$$S_\infty = S = 1$$

Arithmetic Sequences

$$u_1,\ u_1 + d,\ u_1 + 2d,\ u_1 + 3d,\ \ldots$$

d = common difference

Ex 40, 34, 28, 22, 16, 10, ...

(a) Find d: $d = -6$

(b) Find u_{101}: $u_{101} = u_1 + 100d$

$= 40 + 100(-6)$

$= -560$

(c) Find S_{101}:

$$40 + 34 + 28 + \cdots + -548 + -554 + -560$$

-520

-520

-520

$$\left(\frac{101}{2}\right)(-520) = (101)(-260)$$

$$\begin{array}{r} 101 \\ \times\ 260 \\ \hline 2600 \\ \hline -26260 \end{array}$$

number of pairs

value of each pair

Arithmetic Formulas

The n^{th} term: $u_n = u_1 + (n-1)d$

Sum of n terms: $S_n = \left(\frac{n}{2}\right)(u_1 + u_n)$

$$S_n = \left(\frac{n}{2}\right)(u_1 + u_1 + (n-1)d)$$

$$S_n = \left(\frac{n}{2}\right)(2u_1 + (n-1)d)$$

Sigma Notation

$$\sum_{r=1}^{5} r^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

r = the index

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

This is a convergent series
It converges to 1.

1A #5b. $\sum_{r=1}^{10} (-1)^r$

Challenge: Evaluate $\sum_{r=1}^{\infty} (-1)^r = -1+1-1+1-1+1\cdots$

$$(-1+1)+(-1+1)+(-1+1)+(-1+1)\cdots = 0$$

$$-1+(1+-1)+(1+-1)+(1+-1)+(1+-1)+\cdots = -1$$

$\sum_{r=1}^{\infty} (-1)^r$ is a divergent series.

Ex. $\sum_{r=1}^{\infty} r = 1+2+3+\cdots$ is a divergent series

Geometric Sequences

$u_1, u_1r, u_1r^2, u_1r^3, \ldots$

r = common ratio

Ex. $27, 9, 3, 1, \frac{1}{3}, \ldots$

(a) Find r : $r = \frac{1}{3}$

(b) Find u_8 : $u_8 = 27 \cdot \left(\frac{1}{3}\right)^7$

$= \frac{3^3}{3^7} = \frac{1}{3^4} = \frac{1}{81}$

HW 1A #4,5 1B #2-4

1C #1-5 1D #1-6