Antiderivatives For trig functions

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \tan x \, dx = -\int -\frac{\sin x}{\cos x} \, dx$$

$$u = \cos x$$
$$du = -\sin x \, dx$$

$$= -\int \frac{du}{u} = -\ln|u| + C$$

$$= -\ln|\cos x| + C$$

$$= \ln|\cos x|^{-1} + C$$

$$= \ln|\sec x| + C$$

$$\int \tan x \, dx = -\ln|\cos x| + C$$

$$= \ln|\sec x| + C$$

$$\int \cot x \, dx = \int \frac{\cos x}{\sin x} \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$= \int \frac{du}{u} = \ln|u| + C$$

$$= \ln|\sin x| + C$$

$$\int \sec x \, dx$$

$$= \int \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x} dx$$

$$u = \sec x + \tan x$$

$$du = (\sec x \cdot \tan x + \sec^2 x) dx$$

$$= \int \frac{du}{u} = \ln|u| + C$$

$$= \ln|\sec x + \tan x| + C$$

$$\int \csc x \, dx = \int \frac{\csc x(\csc x - \cot x)}{\csc x - \cot x} dx$$

$$u = \csc x - \cot x$$

$$du = (-\csc x \cot x + \csc^2 x) dx$$

$$= \int \frac{du}{u} = \ln|u| + C = \ln|\csc x - \cot x| + C$$

Ex. $\int \frac{e^x}{e^x+1}\,dx = \int \frac{du}{u} = \ln|u| + C$

$u = e^x + 1$

$du = e^x\,dx$

$= \ln(e^x+1) + C$

Ex. $\frac{1}{4}\int \underline{4x^3} \cdot e^{x^4}\,\underline{dx} = \frac{1}{4}\int e^u\,du = \frac{1}{4}e^u + C$

$u = x^4$

$du = \underline{4x^3\,dx}$

$= \frac{1}{4}e^{x^4} + C$

Ex. $\frac{1}{5}\int 5x^4 \cdot \sec x^5\,dx = \frac{1}{5}\int \sec u\,du$

$u = x^5$

$du = 5x^4\,dx$

$= \frac{1}{5}\ln|\sec u + \tan u| + C$

$= \frac{1}{5}\ln|\sec x^5 + \tan x^5| + C$

Ex. $\int x^5 (x^3+1)^6 \, dx$

$$u = x^3 + 1 \Rightarrow x^3 = u - 1$$

$$du = 3x^2 \, dx$$

$$= \frac{1}{3} \int x^3 \cdot \underline{3x^2} \, (x^3+1)^6 \, \underline{dx}$$

$$= \frac{1}{3} \int (u-1) \, u^6 \, du = \frac{1}{3} \int (u^7 - u^6) \, du$$

$$= \frac{1}{24} u^8 - \frac{1}{21} u^7 + C$$

$$= \frac{1}{24} (x^3+1)^8 - \frac{1}{21} (x^3+1)^7 + C$$