① $\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx = -\frac{1}{2}x^{-2} + C$

negative exponents OK for antiderivatives

② $\int \sqrt[3]{x}\,dx = \int x^{1/3}\,dx = \frac{3}{4}x^{4/3} + C$

③ $\int (ax^2 + bx + c)\,dx = \frac{a}{3}x^3 + \frac{b}{2}x^2 + cx + C$

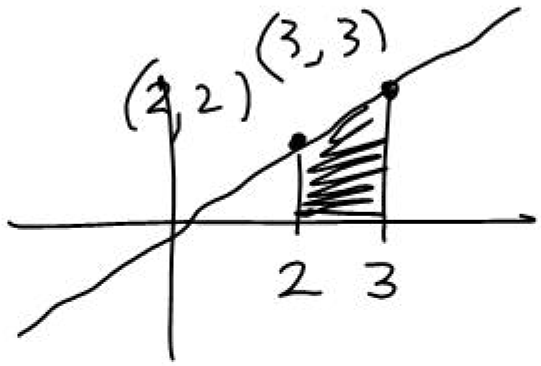
④ $\frac{1}{6}\int 6x\,\overbrace{(3x^2-2)^4}^{\text{composite function}}\,dx = \frac{1}{6}\int u^4\,du$

inner function $= u = 3x^2 - 2$ $\quad = \frac{1}{30}u^5 + C$

$du = \underline{6x\,dx}$ $\quad = \frac{1}{30}(3x^2-2)^5 + C$

$$\frac{d}{dx}\left[x^3\right] = 3x^2$$

$$\int_2^3 x\,dx = 2.5$$



More u-substitution

Ex $\frac{1}{2}\int 2\sqrt{2x-5}\,dx = \frac{1}{2}\int u^{1/2}\,du$

$u = 2x-5$

$du = 2\,dx$

$= \frac{1}{3}u^{3/2} + C$

$= \frac{1}{3}(2x-5)^{3/2} + C$

Ex. $\frac{1}{2}\int \frac{2x}{x^2+1}\,dx$

$u = x^2 + 1$ ← argument (inner function) of the -1 power

$du = 2x\,dx$

$= \frac{1}{2}\int \frac{1}{u}\,du$ OR $\frac{1}{2}\int \frac{du}{u}$

$= \frac{1}{2}\ln|u| + C$ (power rule fails!)

$= \frac{1}{2}\ln(x^2+1) + C$

Ex. $\displaystyle\int \frac{1}{x^2+1}\,dx = \tan^{-1} x + C$

you know the derivative of the function

$\cancel{u = x^2+1}$

$\cancel{du = 2x\,dx}$

Ex. $\displaystyle\int \frac{x^2}{x^2+1}\,dx$

$\cancel{u = x^2+1}$

$\cancel{du = 2x\,dx}$

<u>but</u>

$$
\begin{array}{r@{}l}
 & 1 - \frac{1}{x^2+1} \\
x^2+1 \,\big) & x^2 \\
 & -x^2 - 1 \\
\hline
 & -1
\end{array}
\longrightarrow
$$

$$\int \left(1 - \frac{1}{x^2+1}\right) dx$$

$$= x - \tan^{-1} x + C$$

Ex. $\int x\sqrt{x+1}\,\underline{dx}$

$u = x+1 \longrightarrow x = u-1$

$du = \underline{dx}$

$$= \int (u-1)\,u^{1/2}\,du$$

$\frac{3}{2}+1$

$= \frac{3}{2}+\frac{2}{2}$

$= \frac{5}{2}$

$$= \int \left(u^{3/2} - u^{1/2}\right) du$$

$$= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$$

$$= \underline{\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C}$$