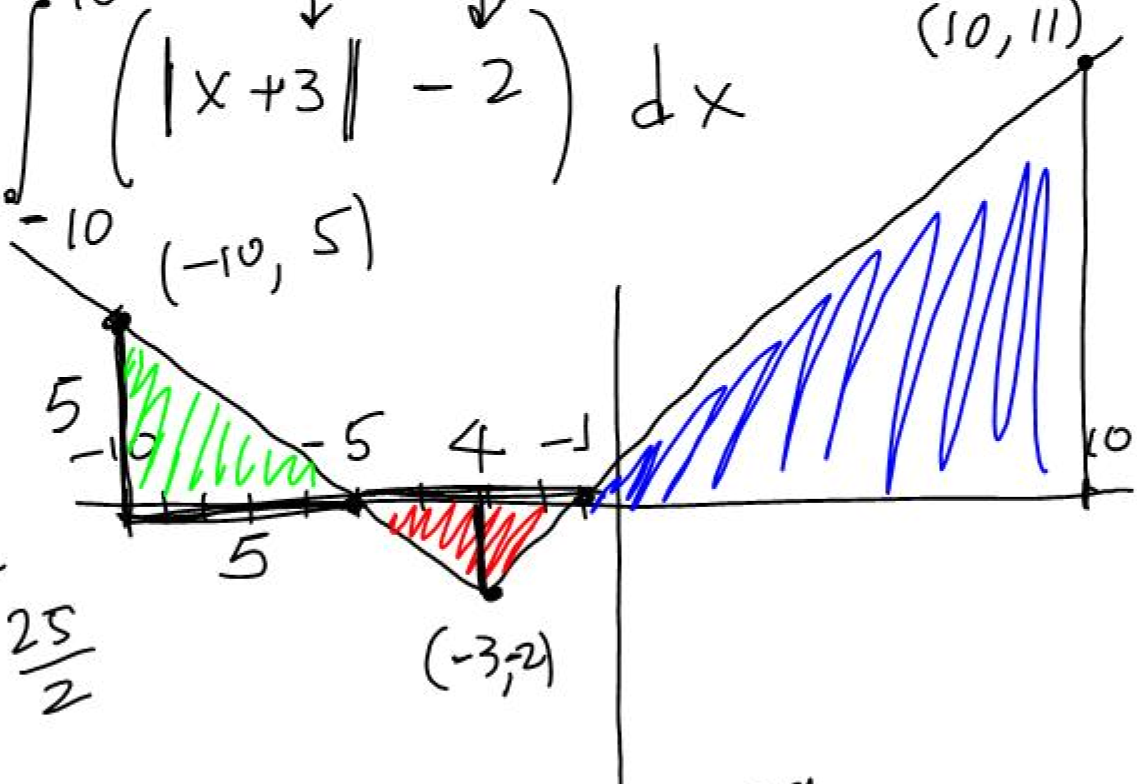
Ex. $\displaystyle\int_{-10}^{10} \left(\overset{\text{left}}{|x+3|} - \overset{\text{down}}{2} \right) dx$



green area

$\frac{1}{2}(5)(5) = \frac{25}{2}$

red area

$\frac{1}{2}(4)(-2) = -4$

blue area

$\frac{1}{2}(11)(11) = \frac{121}{2}$

$$\int_{-10}^{10} \left(|x+3| - 2 \right) dx = \frac{25}{2} - \frac{8}{2} + \frac{121}{2} = 69$$

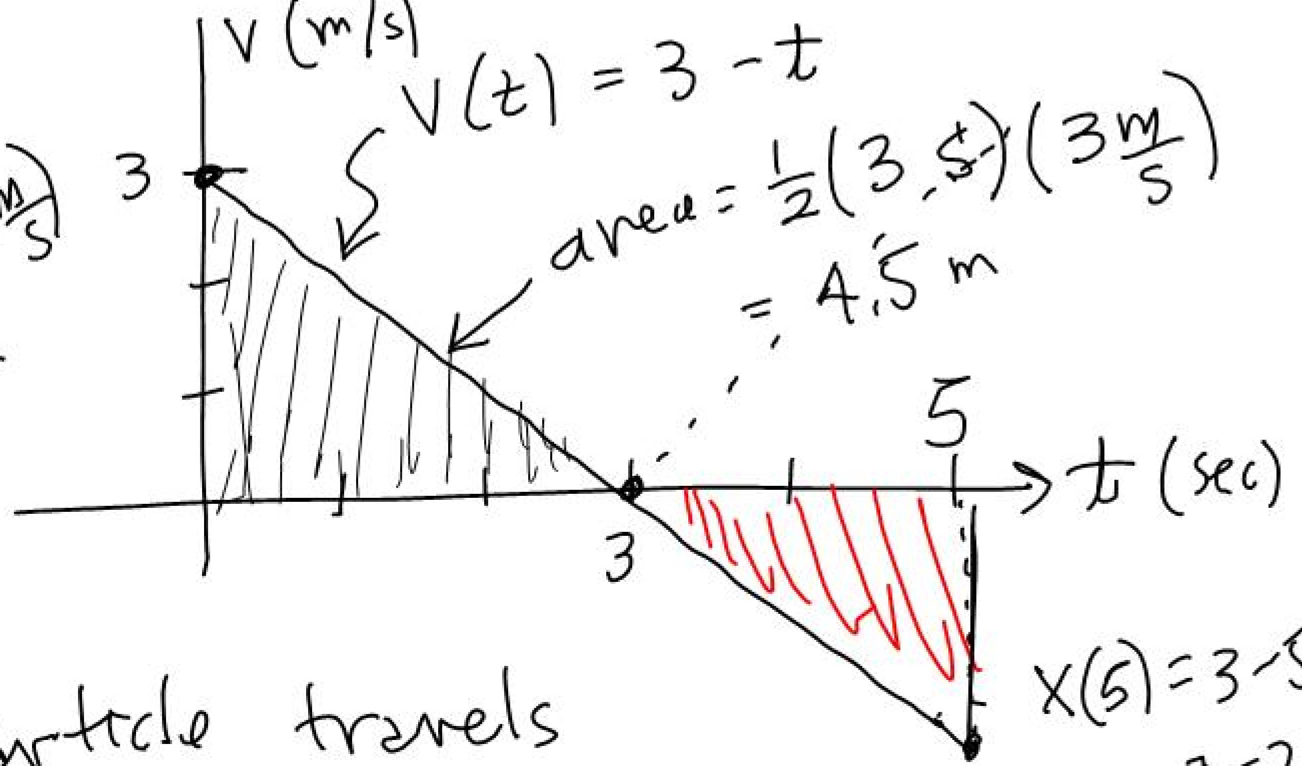
Ex. $$\int_{10}^{-10} \left(|x+3| - 2 \right) dx = -\frac{25}{2} + \frac{8}{2} - \frac{121}{2} = -69$$

Ex.

red area

$\frac{1}{2}(2s)\left(-2\frac{m}{s}\right)$

$= -2\,m$



A particle travels along the x-axis.

Its ~~distance from the origin~~ velocity is given by $v(t) = 3 - t$.

Find the <u>displacement</u> of the particle for $0 \le t \le 5$.

$$\int_0^5 (3-t)\,dt = 4.5 - 2 = 2.5\ m$$

to the right

$$\int_{t_1}^{t_2} v(t)\,dt = \text{displacement on the time interval } [t_1, t_2].$$

$$\int_{t_1}^{t_2} |v(t)|\, dt = \text{distance travelled}$$

HW quiz

$$\int_0^5 \left(|x-2| - 1 \right) dx$$

Antiderivatives

a.k.a. Indefinite Integrals

$$\int \underset{\text{derivative}}{\cos x}\, dx = \underset{\text{original function}}{\sin x} + C$$

C = constant of integration

Ex. $\int (x^2 - 3x + 2)\, dx$

$$= \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x + C$$

The Power Rule (for antiderivatives)

$$\boxed{\int x^n\, dx = \frac{1}{n+1}x^{n+1} + C, \quad n \neq -1}$$

Ex $\int \sqrt{x}\,dx = \int x^{1/2}\,dx$

$= \frac{2}{3}x^{3/2} + C$

Ex $\int \frac{1}{x}\,dx = \ln|x| + C$ *

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u-substitution

Ex. $\frac{1}{2}\int \frac{2x}{x^2+3}\,dx = \frac{1}{2}\int \frac{1}{u}\,du$

$u = x^2 + 3$

$\frac{du}{\cancel{dx}} = 2x\,dx$

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln(x^2+3) + C$

check

$\frac{d}{dx}\left[\frac{1}{2}\ln(x^2+3)\right] = \frac{1}{\cancel{2}} \cdot \frac{1}{x^2+3} \cdot \cancel{2}x$

$$\frac{1}{12}\int \underline{12x^2}\,\overbrace{(4x^3-10)^{12}}^{\text{composite}}\,\underline{dx}$$

inner function $\longrightarrow$ $u = 4x^3 - 10$

$$du = \underline{12x^2\,dx}$$

$\frac{13}{12}$

$$= \frac{1}{12}\int u^{12}\,du = \frac{1}{156}u^{13} + C$$

$$= \underline{\frac{1}{156}(4x^3-10)^{13} + C}$$

Exercises

① $\displaystyle\int \frac{1}{x^3}\,dx$ ② $\displaystyle\int \sqrt[3]{x}\,dx$

③ $\displaystyle\int (ax^2+bx+c)\,dx$

④ $\displaystyle\int x(3x-2)^4\,dx$