#25 f

$$e^{2x} = 1 + 2x + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} \cdots$$

$$xe^{2x} \approx x + 2x^2 + \frac{2^2x^3}{2!} + \frac{2^3x^4}{3!} + \frac{2^4x^5}{4!} + \frac{2^5x^6}{5!} + \frac{2^6x^7}{6!}$$

(g) $$e^{x^2} = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} \cdots$$

$$x^3e^{x^2} = x^3 + x^5 + \frac{x^7}{2!} + \frac{x^9}{3!} \cdots$$

$+x+2$ $+x+2$

$$x^3e^{x^2} + x + 2 \approx 2 + x + x^3 + x^5 + \frac{x^7}{2!}$$

#26 (a) $\sin x \approx x - \frac{x^3}{3!}$

(b) $\frac{\pi}{6} - \frac{(\pi/6)^3}{3!} = 0.49967$ error 0.063%

$\sin\frac{\pi}{6} = \frac{1}{2}$

(c) $\sin x \approx \underbrace{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}}_{P_7(x)}$

$P_7\left(\frac{\pi}{6}\right) = 0.499999919$

1.6×10^{-6} % error

(d)

(e) $\sin x^2 \approx x^2 - \frac{x^6}{3!}$

(f) $\underbrace{x^2}_{\text{even}} \cdot \underbrace{\sin x}_{\text{odd}} \approx x^3 - \frac{x^5}{3!} + \frac{x^7}{5!}$

odd: $\sin(-x) = -\sin x$

even: $\cos(-x) = \cos x$

(g) $4 + \sin x = 4 + x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots$

$$\frac{4+\sin x}{x} = \frac{4}{x} \sim 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \frac{x^8}{9!} \cdots$$

$$\boxed{\frac{1}{x-3} \approx 1 - (x-4) + (x-4)^2 - (x-4)^3 + (x-4)^4}$$

#27

n	$f^{(n)}(x)$	$f^{(n)}(4)$	$\frac{f^{(n)}(4)}{n!}$
0	$(x-3)^{-1}$	1	1
1	$-(x-3)^{-2}$	-1	-1
2	$2(x-3)^{-3}$	2	1
3	$-6(x-3)^{-4}$	-6	-1
4	$24(x-3)^{-5}$	24	1

HW quiz

(1) Write $P_4(x)$ for $f(x) = \dfrac{1}{x-5}$ centered at $x = 4$.

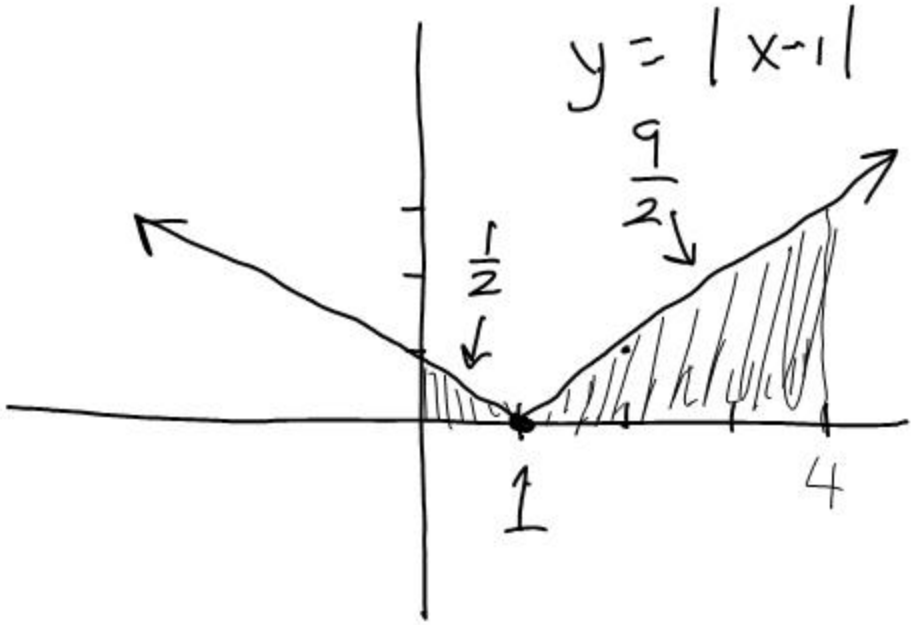
(2) Write P_6 for $f(x) = \cos x$ centered at $x = 0$

(3) Write P_6 for $f(x) = x^2 \cdot \cos x - x$ centered at $x = 0$

Definite Integrals

$$\int_0^4 |x-1|\,dx$$

- 4: upper end point
- $|x-1|$: integrand (the function we're integrating)
- dx: differential term
- 0: lower end point
- $\int$: integral sign

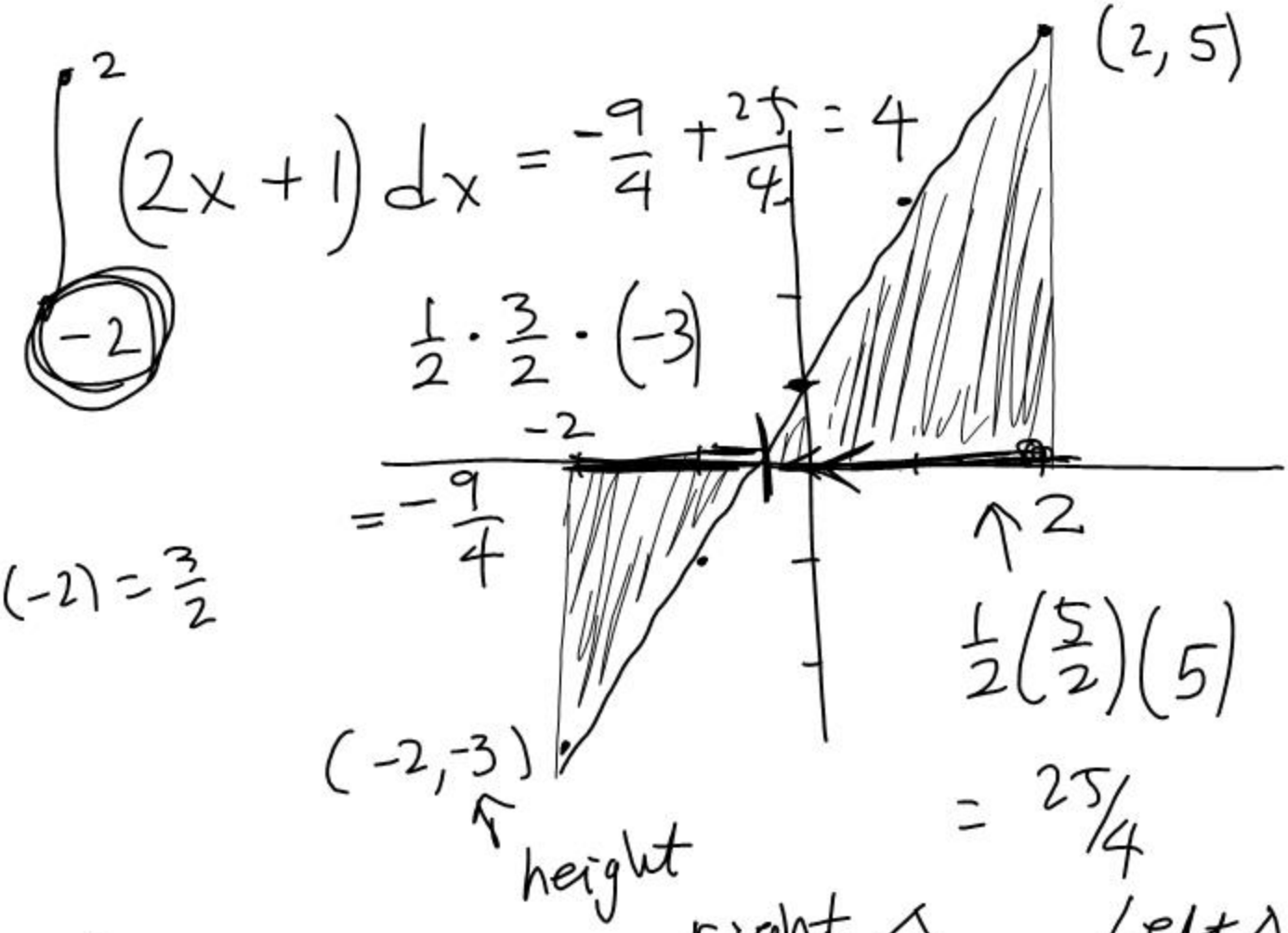


$$\int_0^4 |x-1|\,dx = 5$$

$$\frac{d}{dx}[x^3] = 3x^2$$

$$\frac{d}{dt}[x^3] = 3x^2 \cdot \frac{dx}{dt}$$

$\int_{-2}^{2} (2x+1)\,dx = -\frac{9}{4} + \frac{25}{4} = 4$



$\frac{1}{2} \cdot \frac{3}{2} \cdot (-3)$

$= -\frac{9}{4}$

$-\frac{1}{2} - (-2) = \frac{3}{2}$

$\frac{1}{2}\left(\frac{5}{2}\right)(5)$

$= 25/4$

right $\triangle$ Left $\triangle$

$\int_{2}^{-2} (2x+1)\,dx = \frac{1}{2}\left(\frac{-5}{2}\right)(5) + \frac{1}{2}\left(\frac{-3}{2}\right)(-3)$

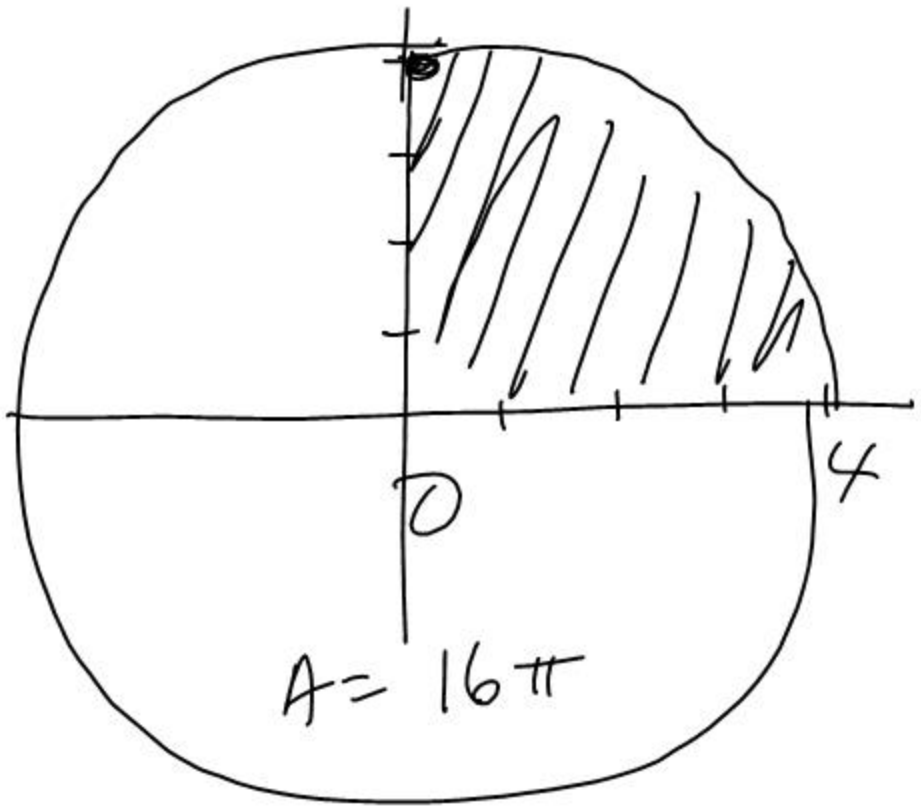
$= \frac{-25}{4} + \frac{9}{4} = -4$

Ex. $\int_0^4 \sqrt{16-x^2}\,dx = 4\pi$

$y = \sqrt{16-x^2}$

$y^2 = 16-x^2$

$x^2+y^2 = 16$



Exercises

① $\int_{-1}^{3} x\,dx$

② $\int_{-3}^{1} |x+1|\,dx$

③ $\int_{1}^{-1} \sqrt{1-x^2}\,dx$