#17

n	$f^{(n)}(x)$	$f^{(n)}(4)$	$\frac{f^{(n)}(4)}{n!}$
0	$x^{1/2}$	2	2
1	$\frac{1}{2}x^{-1/2}$	$\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$	$\frac{1}{4}$
2	$-\frac{1}{4}x^{-3/2}$	$-\frac{1}{4}\cdot\frac{1}{8}=\frac{-1}{32}$	$-\frac{1}{64}$
3	$\frac{3}{8}x^{-5/2}$	$\frac{3}{8}\cdot\frac{1}{32}=\frac{3}{256}$	$\frac{3}{256}\cdot\frac{1}{6}=\frac{1}{512}$

$$\sqrt{x} \approx 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 + \frac{1}{512}(x-4)^3$$

$$4^{-5/2} = \left(4^{1/2}\right)^{-5} = 2^{-5} = \frac{1}{32}$$

#18

n	$f^{(n)}(x)$	$f^{(n)}(2)$	$\frac{f^{(n)}(2)}{n!}$
0	$(x-1)^{-1}$	1	1
1	$-(x-1)^{-2}$	-1	-1
2	$2(x-1)^{-3}$	2	1
3	$-6(x-1)^{-4}$	-6	-1

$$\frac{1}{x-1} \approx 1-(x-2)+(x-2)^2-(x-2)^3$$

#19 $f(x)=\frac{1}{x-4}$, $x=3$

n	$f^{(n)}(x)$	$f^{(n)}(3)$	$\frac{f^{(n)}(3)}{n!}$
0	$(x-4)^{-1}$	-1	-1
1	$-(x-4)^{-2}$	-1	-1
2	$2(x-4)^{-3}$	-2	-1
3	$-6(x-4)^{-4}$	-6	-1

$$\frac{1}{x-4} \approx -1 - (x+3) - (x+3)^2 - (x+3)^3$$

#20

n	$f^{(n)}(x)$	$f^{(n)}(1)$	$\frac{f^{(n)}(1)}{n!}$
0	x^{-1}	1	1
1	$-x^{-2}$	-1	-1
2	$2x^{-3}$	2	1
3	$-6x^{-4}$	-6	-1

$$\frac{1}{x} \approx 1 - (x-1) + (x-1)^2 - (x-1)^3$$

#24

(a) $f(x) = (1+x)^{-1}$ $\quad f(0) = 1$

$f'(x) = -(1+x)^{-2}$ $\quad f'(0) = -1$

$$\boxed{P_1(x) = 1 - x}$$

(b) $P_1(0.25) = 0.75$

$f(0.25) = \dfrac{1}{1+0.25} = \dfrac{1}{5/4} = \dfrac{4}{5} = 0.8$

$\%\text{ error} = \dfrac{|0.8 - 0.75|}{0.8} \times 100 = 6.25\%$

(c) $f''(x) = 2(1+x)^{-3}$ $\quad f''(0) = 2$

$$\boxed{P_2(x) = 1 - x + x^2}$$

(d) $P_2(0.25) = 1 - \frac{1}{4} + \frac{1}{16} = \frac{13}{16}$

$\%\text{ error} = \dfrac{\left|\frac{4}{5} - \frac{13}{16}\right|}{\frac{4}{5}} \times 100 = 1.6\%$

(e) $f'''(x) = -6(1+x)^{-4}$

$f'''(0) = -6$

$$\boxed{P_3(x) = 1 - x + x^2 - x^3}$$

(f) $P_3(0.25) = \frac{51}{64}$

$$\%\ \text{error} = \frac{\left|\frac{51}{64} - \frac{4}{5}\right| \times 100}{\frac{4}{5}} = 0.4\%$$

(g) $P_8(x) = 1 - x + x^2 - x^3 + x^4 - x^5 + x^6 - x^7 + x^8$

(h) $P_8(0.25) = 0,800003051758$

$\%$ error $= 0.0004\%$

(i) $P_9 = 1 - x^3 + (x^3)^2 - (x^3)^3 \approx \frac{1}{1+x^3}$

$$\boxed{P_9 = 1 - x^3 + x^6 - x^9}$$

(j) $P_{11}(x) = x^2 - x^5 + x^8 - x^{11}$

(k) $P_4(x) = 1 - (-x) + (-x)^2 - (-x)^3 \cdots$

$$\frac{1}{1-x} \approx P_4(x) = 1 + x + x^2 + x^3 + x^4$$

(l) $$\frac{1}{1-x^2} \approx 1 + x^2 + x^4 + x^6 + x^8$$
$$(x^2)^2 + (x^2)^3 + (x^2)^4$$

$$\frac{x}{1-x^2} \approx x + x^3 + x^5 + x^7$$

#25 (a) $P_5(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}$

(b) $P_5(x) = 1 + (2x) + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} + \frac{(2x)^5}{5!}$

(c) $P_{10}(x) = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \frac{x^{10}}{5!}$

(d)

$$e^x - 3x - 4 \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \cdots$$

$$e^x - 3x - 4 \approx -3 - 2x + \frac{x^2}{2!} + \frac{x^3}{3!} \cdots$$

$$\frac{e^x - 3x - 4}{x} \approx -\frac{3}{x} - 2 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \frac{x^4}{5!} + \frac{x^5}{6!}$$

(e) $$e^{2x} \approx 1 + (2x) + \frac{(2x)^2}{2!} + \cdots$$

$$e^{2x} - 2x - 3 \approx -2 + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} \cdots$$

$$\frac{e^{2x} - 2x - 3}{x^2} \approx \frac{-2}{x^2} + \frac{2^2}{2!} + \frac{2^3 x}{3!} + \frac{2^4 x^2}{4!} + \frac{2^5 x^3}{5!} + \frac{2^6 x^4}{6!} + \frac{2^7 x^5}{7!}$$