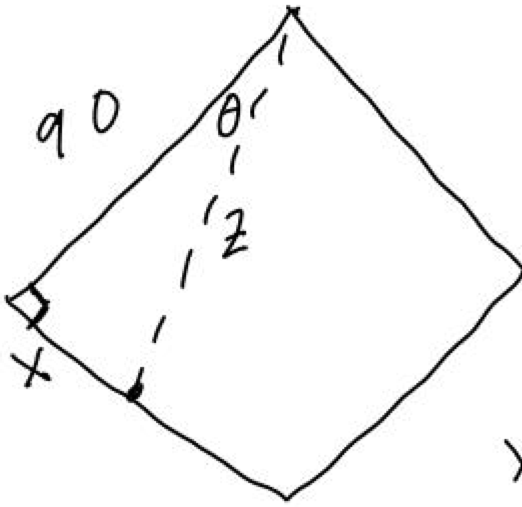
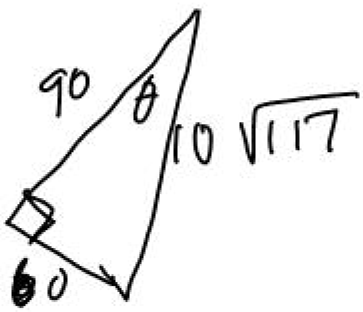
11



$$x^2 + 90^2 = z^2$$

$$\cancel{2}x\frac{dx}{dt} = \cancel{2}z\frac{dz}{dt}$$



$$60(28) = 10\sqrt{117}\,\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{6\cancel{0}\cdot 28}{\cancel{10}\sqrt{117}} \text{ ft/sec}$$

$$= \frac{168}{\sqrt{117}} \text{ ft/sec}$$

$$\approx 15.5 \text{ ft/sec}$$

(b) $\frac{d\theta}{dt} = ?$ $\quad \frac{dx}{dt} = 28$

$$\tan\theta = \frac{x}{90}$$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{90}\frac{dx}{dt}$$

$$\left(\frac{10\sqrt{117}}{90}\right)^2 \cdot \frac{d\theta}{dt} = \frac{1}{90}\cdot 28$$

$$\frac{d\theta}{dt} = \frac{\overset{14}{\cancel{28}}\cdot \overset{2}{\cancel{90}}}{\cancel{90}\cdot \underset{5}{\cancel{100}}\cdot \underset{13}{\cancel{117}}} = \frac{126}{585} = \frac{14}{65}$$

$$\boxed{\begin{array}{l}\frac{14}{65} \text{ rad/sec} \\ = 12.3^\circ \text{ deg/sec}\end{array}}$$

$$\frac{14}{65}\cdot\frac{180^\circ}{\pi}$$

$\cancel{\text{rad}}$
sec

#15 (a) $h = 2r$

$$V = \pi r^2 h = \pi r^2(2r)$$

$$V = 2\pi r^3$$

$$\boxed{\frac{dV}{dt} = 6\pi r^2 \frac{dr}{dt}}$$

$$16\cancel{\pi} = 2\cancel{\pi} r^3$$
$$8 = r^3$$
$$2 = r$$
$$4 = h$$

$$\left.\frac{dV}{dt}\right|_{V=16\pi} = 6\pi(2)^2 \cdot 10 = 240\pi \ \mathrm{cm^3/s}$$

(b) $\frac{dV}{dt} = 10$

$$V = \pi\left(\frac{h}{2}\right)^2 h = \frac{\pi}{4}h^3$$

$$\frac{dV}{dt} = \frac{3\pi}{4}h^2 \cdot \frac{dh}{dt}$$

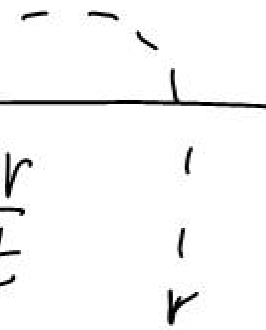
$$10 = \frac{3\pi}{\cancel{4}} \cdot 4^{\cancel{2}} \cdot \frac{dh}{dt}$$

$$\frac{10}{12\pi} = \frac{dh}{dt} = \frac{5}{6\pi} \ \mathrm{cm/s}$$

$$V = \underbrace{\pi r^2}\,\underbrace{h}$$

$$h = 2r$$

$$\frac{dh}{dt} = 2\frac{dr}{dt}$$



$$\frac{dV}{dt} = \pi r^2 \cdot \frac{dh}{dt} + h \cdot 2\pi r \frac{dr}{dt}$$

$$10 = \pi\, 2^2 \cdot \frac{dh}{dt} + 4 \cdot 2\pi \cdot \cancel{2} \cdot \left(\frac{1}{\cancel{2}}\frac{dh}{dt}\right)$$

$$10 = \frac{dh}{dt}\left(4\pi + 8\pi\right) \Rightarrow \frac{dh}{dt} = \frac{10}{12\pi} \checkmark$$

(c) $\frac{dh}{dt} = 5$ cm/s $\qquad h = 2r$

$$A = 2\pi r^2 + 2\pi r \cdot h$$

$$A = 2\pi \cdot \frac{h^2}{4} + 2\pi \cdot \left(\frac{h}{2}\right) \cdot h = \frac{\pi}{2} h^2 + \pi h^2$$

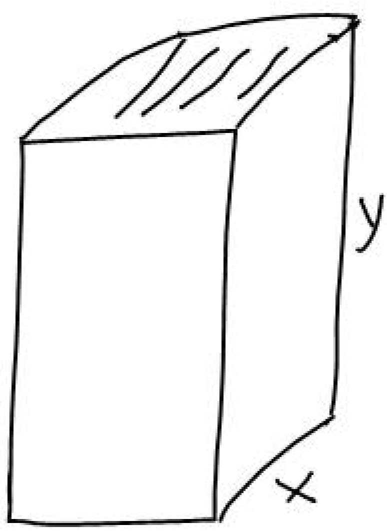
$$A = \frac{3\pi}{2} h^2$$

$$\frac{dA}{dt} = 3\pi\, h \cdot \frac{dh}{dt}$$

$$\left.\frac{dA}{dt}\right|_{V=16\pi} = 3\pi \cdot 4 \cdot 5$$

$$= 60\pi \ \text{cm}^2/\text{s}$$

Ex. You are building a square bottom open-top rectangular box. Material for the bottom costs \$1/ft^2, for the sides \$0.40/ft^2. If the box contains 4 ft^3, find the cheapest way to build.



$$C = 1x^2 + 0.4(4xy)$$

$$V = x^2 y = 4$$

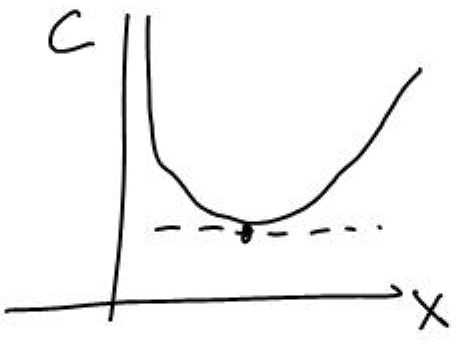
$$y = \frac{4}{x^2}$$

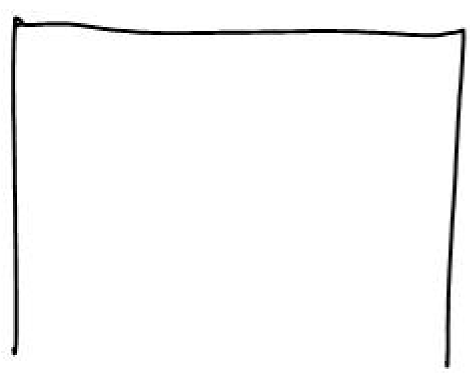
$$16 \times .4 = 6.4$$

$$C = x^2 + 6.4x^{-1}$$

$$\frac{dC}{dx} = C' = 2x - 6.4x^{-2} = 0$$

$$2x^3 - 6.4 = 0$$





$$x^3 = 3.2$$

$$x \approx 1.47 \text{ ft}$$

Ex. $\lim_{x\to 0}(1+x)^{1/x} = e^1$ $\qquad 5^x = 10$

$\boxed{1^\infty}$

New problem:

$\boxed{\ln a^b = b\cdot \ln a}$

$$\lim_{x\to 0} \ln\left((1+x)^{1/x}\right)$$

$$= \lim_{x\to 0} \frac{1}{x}\cdot \ln(1+x) = \lim_{x\to 0}\frac{\ln(1+x)}{x} \leftarrow \boxed{\frac{0}{0}}$$

$$\stackrel{L'H}{=} \lim_{x\to 0}\frac{\frac{1}{1+x}\cdot 1}{1} = 1$$

L'Hôpital's Rule

Ex. $\lim_{x\to\infty} x\cdot\sin\frac{1}{x} = \lim_{x\to\infty}\frac{\sin\frac{1}{x}}{\frac{1}{x}}$

$\boxed{\infty\cdot 0}$ $\qquad \boxed{\frac{0}{0}}$

$$\stackrel{L'H}{=} \lim_{x\to\infty}\frac{\cos\frac{1}{x}\cdot\cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}} = 1$$

$$\lim_{x\to\infty}\frac{1}{x} = 0$$