

Related Rates

#1. (a) $A = \pi r^2$

$$\frac{d}{dt}[A] = \frac{d}{dt}[\pi r^2]$$

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

$\text{ft}^2/\text{min} = \text{ft} \cdot \frac{\text{ft}}{\text{min}}$

(b) $25 = 2 \cdot \pi \cdot 20 \cdot \frac{dr}{dt}$

$$\frac{dr}{dt} = \frac{5}{8\pi} \text{ ft/min}$$

(c) $\frac{dr}{dt} = \frac{25}{80\pi} = \frac{5}{16\pi} \text{ ft/min}$

(d) $C = 2\pi r$

$$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$$

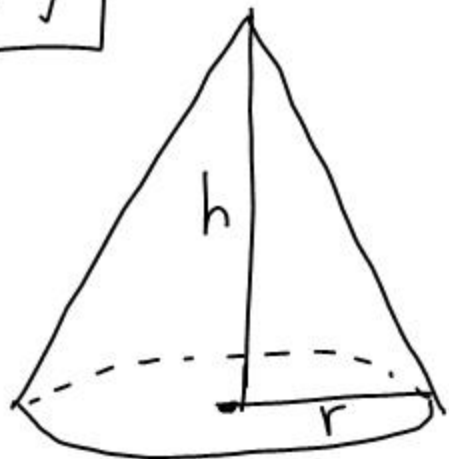
?

(e) $\left. \frac{dr}{dt} \right|_{C=80\pi} = ?$

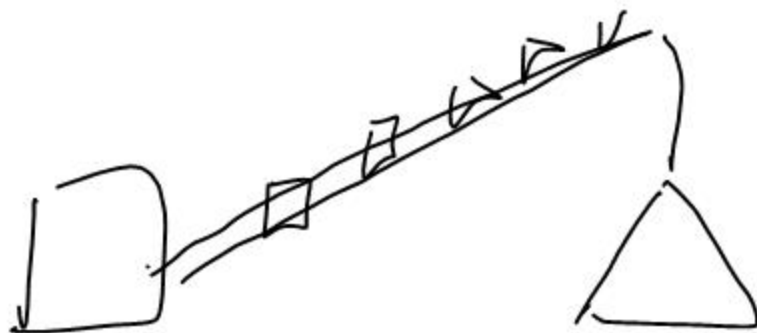
$$80\pi = 2\pi r \Rightarrow r = 40$$

$$\left. \frac{dC}{dt} \right|_{C=80\pi} = 2\pi \left(\frac{5}{16\pi} \right) = \frac{5}{8} \text{ ft/min}$$

#7



$$h = 3r$$



$$\frac{dV}{dt} = 1 \text{ m}^3/\text{s}$$

$$\begin{aligned} \text{(a)} \quad V &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \pi r^2 (3r) \end{aligned}$$

$$V = \pi r^3$$

$$\text{(b)} \quad \frac{dV}{dt} = 3\pi r^2 \frac{dr}{dt}$$

$V = \pi r^3$
 $64\pi = \pi r^3$

$r = 4 \text{ m}$

$$1 = 3\pi(4)^2 \cdot \frac{dr}{dt}$$

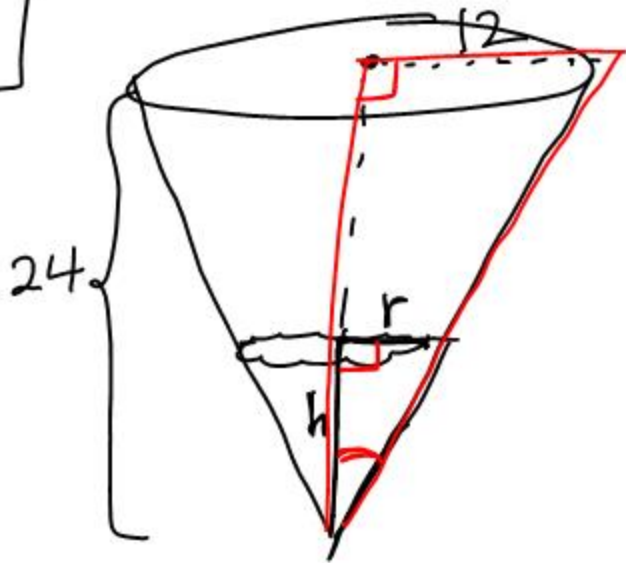
$$\frac{dr}{dt} = \frac{1}{48\pi} \frac{\text{m}}{\text{s}}$$

$$(c) \quad h = 3r$$

$$\frac{dh}{dt} = 3 \cdot \frac{dr}{dt}$$

$$\left. \frac{dh}{dt} \right|_{V=64\pi} = \frac{3}{48\pi} = \frac{1}{16\pi} \text{ m/s}$$

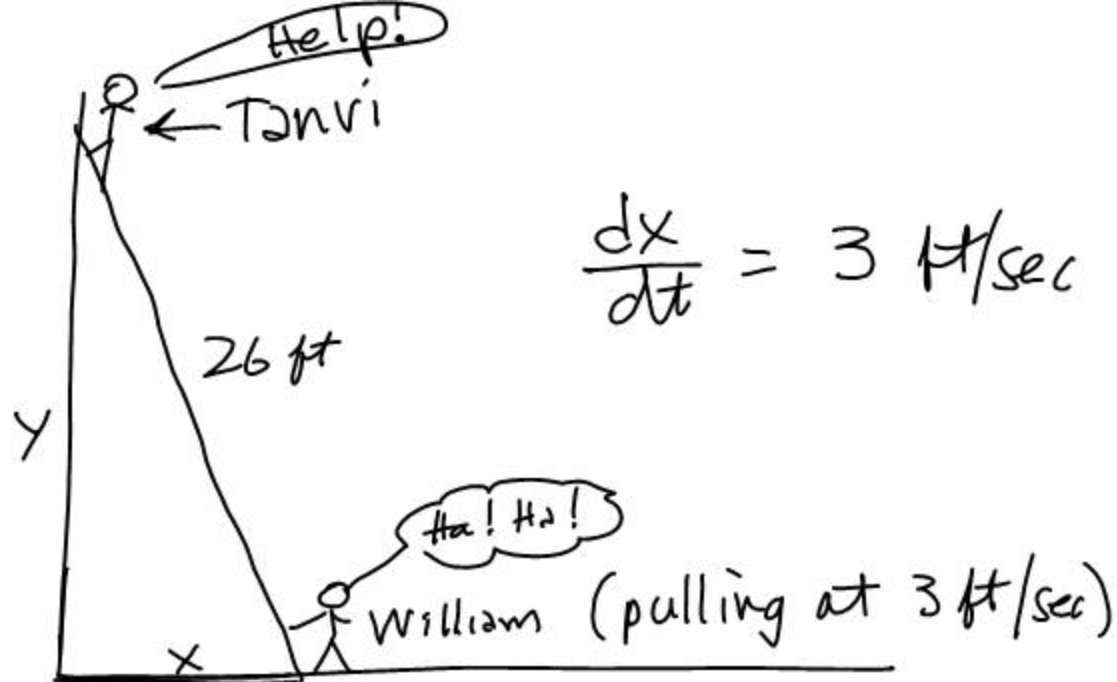
#8



$$\frac{r}{12} = \frac{h}{24}$$

$$2r = h$$

#13



5, 12, 13
10, 24, 26

(a) $\left. \frac{dy}{dt} \right|_{x=10} = ?$

$$x^2 + y^2 = 26^2$$

$$\cancel{x} \frac{dx}{dt} + \cancel{y} \frac{dy}{dt} = 0$$

(b) $\left. \frac{dy}{dt} \right|_{x=24} = ?$

$$10 \cdot (3) + (24) \frac{dy}{dt} = 0$$

$$24(3) + 10 \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-30}{24}$$

$$\frac{dy}{dt} = -\frac{36}{5} \text{ ft/sec}$$

$$= \frac{-5}{4} \text{ ft/sec}$$

$$x \frac{dx}{dt} + y \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-x \frac{dx}{dt}}{y} \rightarrow \text{increasing}$$

$$(c) A = \frac{1}{2}xy$$

$$\frac{dA}{dt} = \frac{1}{2}x \cdot \frac{dy}{dt} + y \cdot \frac{1}{2} \frac{dx}{dt}$$

$$x^2 + y^2 = 26^2$$

$$y = \sqrt{26^2 - x^2}$$

$$\left. \frac{dA}{dt} \right|_{x=10} = \frac{1}{2} \left(\frac{5}{10} \right) \left(-\frac{5}{4} \right) + 24 \cdot \frac{1}{2} \cdot (3)$$

$$= \frac{-25}{4} + 36 = \frac{119}{4} \text{ ft}^2/\text{sec}$$

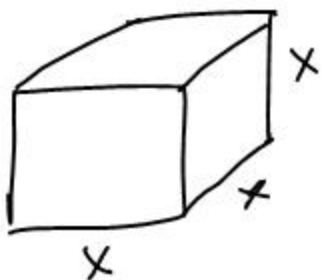
Spheres

$$V = \frac{4}{3} \pi r^3$$

(surface) $A = 4 \pi r^2$

~~circles~~
 $A = \pi r^2$
 $C = 2 \pi r$

Cubes



$$V = x^3$$

$$A = 6x^2$$

edges: 12
vertices: 8

$$E = F + V - 2$$

Cone : $V = \frac{1}{3} \pi r^2 h$

90 ft, $60\frac{1}{2}$ ft, 18 ft, —

HW # 2, 5, 8, 9, 11
