① $\dfrac{2x}{x^2+4}$

② $\dfrac{4}{\sqrt{1-(4x)^2}}$

$= \dfrac{4}{\sqrt{1-16x^2}}$

③ $\dfrac{2x}{1+(x^2)^2}$

$= \dfrac{2x}{1+x^4}$

④ $\dfrac{\cos t}{\sin t} = \cot t$

⑤ $2^t \cdot \ln 2$

e^t

⑥ $\dfrac{t \cdot \frac{1}{t} - \ln t}{t^2}$

$= \dfrac{1-\ln t}{t^2}$

⑦ $\underline{(x \cdot e^y \cdot y'} + e^y) - (ye^x + \underline{e^x \cdot y'}) = 0$

$y'\cancel{(xe^y - e^x)} = \dfrac{ye^x - e^y}{xe^y - e^x}$

$$(x \cdot \sec y \tan y \cdot y' + \sec y) + (y \cdot \sec^2 x + \tan x \cdot y') = 0$$

$$y'(x \cdot \sec y \tan y + \tan x) = -\sec y - y\sec^2 x$$

$$y' = \frac{-\sec y - y\sec^2 x}{x\sec y \tan y + \tan x}$$

$$3x^2 - (3x^2 \cdot y' + y \cdot 6x) + (y \cdot 2x + x^2 \cdot y') - 3y^2 y' = 0$$

$$y'(\overset{-2x^2}{\cancel{-3x^2 + x^2}} - 3y^2) = -3x^2 + \overset{4xy}{\cancel{6xy - 2xy}}$$

$$y' = \frac{-4xy + 3x^2}{+2x^2 + 3y^2}$$

$$\cosh x = \frac{e^{x}+e^{-x}}{2}$$

$\cos 0 = 1$

$\cosh 0 = 1$

$$\operatorname{sech} x = \frac{2}{e^{x}+e^{-x}}$$

$$\frac{d}{dx}\left[\frac{2}{e^{x}+e^{-x}}\right] = \frac{0-2(e^{x}-e^{-x})}{(e^{x}+e^{-x})^{2}}$$

$$= \frac{-2}{e^{x}+e^{-x}} \cdot \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$

$$= -\operatorname{sech} x \cdot \tanh x$$

$$= \frac{d}{dx}\left[\operatorname{sech} x\right]$$

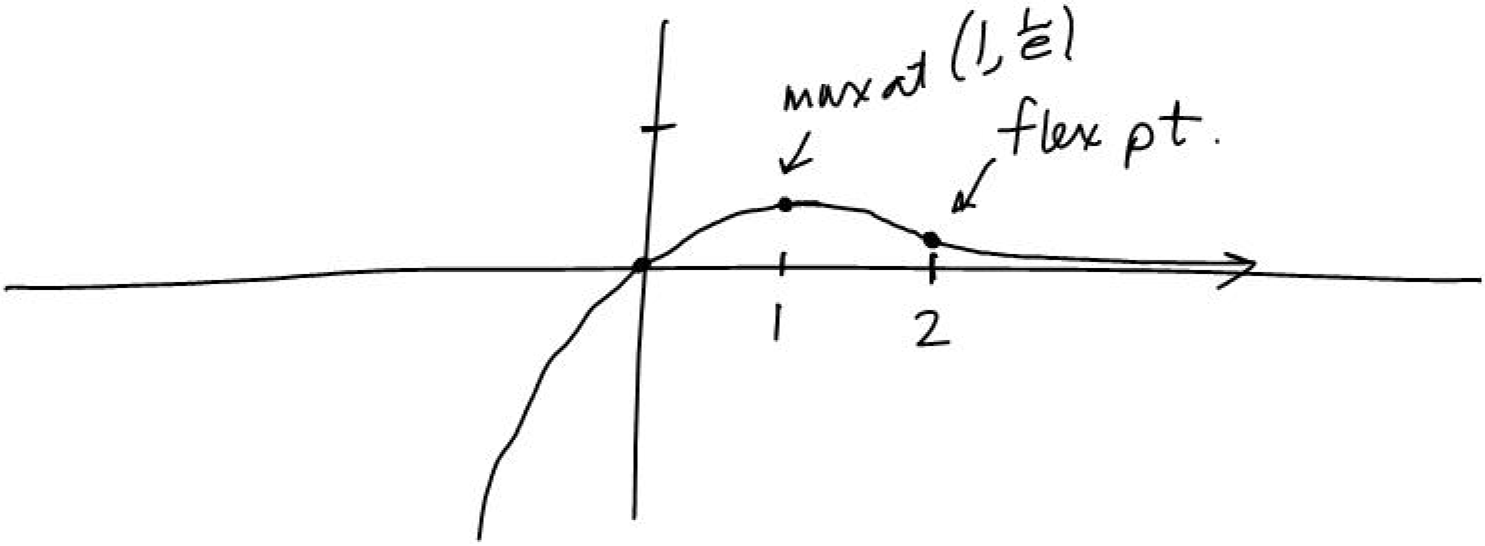
$$f(x) = \frac{x}{e^x}$$

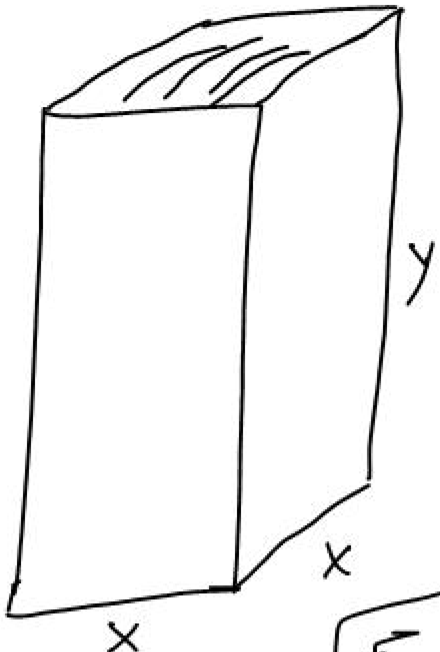
$$f'(x) = \frac{e^x - xe^x}{e^{2x}} = \frac{1-x}{e^x}$$

$f'(x)$: + | − ; incr | 1 | decr

$$f''(x) = \frac{\cancel{e^x}(-1) - (1-x)\cancel{e^x}}{(e^x)^{\cancel{2}}} = \frac{x-2}{e^x}$$

$f''(x)$: − | + ; concave down | 2 | concave up





Ex. An open-top rectangular box must contain 3 ft^3. The bottom is square. Find the minimum surface area.

Function : $A = x^2 + 4xy$

equation : $x^2 y = 3$

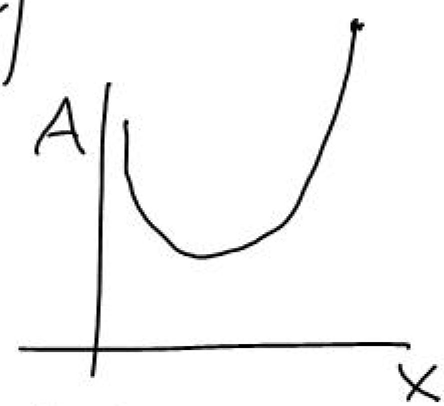
$$y = \frac{3}{x^2}$$

$$A = x^2 + 4x\left(\frac{3}{x^2}\right)$$

$12x^{-1}$

$-12x^{-2}$

$$\boxed{A = x^2 + \frac{12}{x}}$$



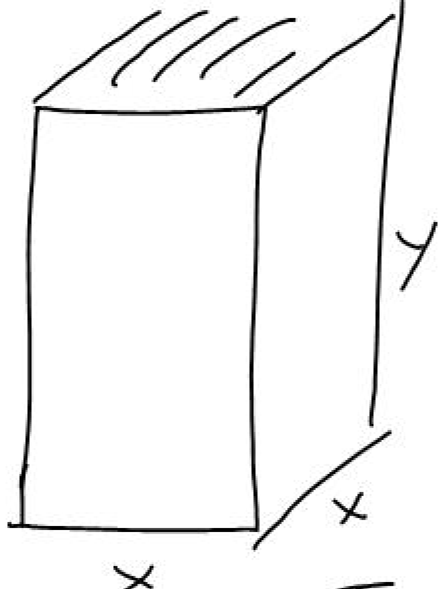
Derivative : $A'(x) = 2x - \frac{12}{x^2} = 0$

$$2x^3 - 12 = 0$$

$$x = \sqrt[3]{6} \text{ ft}$$

Min. area is

$6^{2/3} + \frac{12}{6^{1/3}}$ ft^2



An open-top box with a square bottom is to use 9 ft^2 of material. Find the maximum volume.

Function: $V = x^2 y$

equation: $x^2 + 4xy = 9$

$$4xy = 9 - x^2$$

$$y = \frac{9 - x^2}{4x}$$

Function: $V = x^2\left(\frac{9-x^2}{4x}\right) = \frac{9x - x^3}{4}$

Derivative: $V' = \frac{9 - 3x^2}{4} = 0$

$x = \sqrt{3}$ ft

Max. volume is

$$V = \frac{9\sqrt{3} - 3\sqrt{3}}{4} = \frac{3\sqrt{3}}{2} \text{ ft}^3$$