#3 $f(x) = x^3 - 4x^2 - 3x + 18$

$f'(x) = 3x^2 - 8x - 3 = 0$

$= (3x+1)(x-3) = 0$

critical values $\rightarrow x = -1/3$ or 3

$f'(x)$: $\quad + \quad \underset{-\frac{1}{3}}{|} \quad - \quad \underset{3}{|} \quad +$

f is increasing on $(-\infty, -\frac{1}{3}) \cup (3, \infty)$

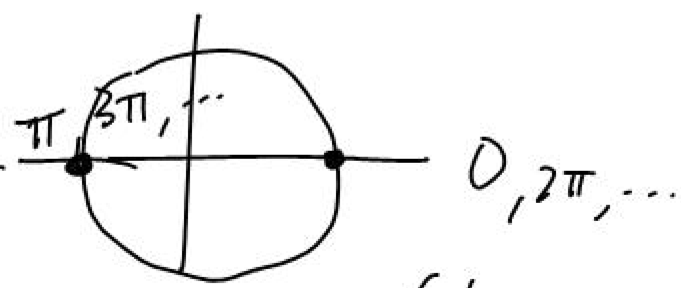
max at $x = -1/3$

min at $x = 3$

#4. $f(x) = \sin^2 x \;, \; 0 \le x \le 2\pi$

$f'(x) = 2\sin x \cdot \cos x = \sin 2x = 0$

$2x = 0, \pi, 2\pi, \ldots$



$\boxed{x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi}$

$f'(x)$

increasing $f'(x)$: $\underset{0}{|} \quad + \quad \underset{\pi/2}{|} \quad - \quad \underset{\pi}{|} \quad + \quad \underset{\frac{3\pi}{2}}{|} \quad - \quad \underset{2\pi}{|}$

$(0, \frac{\pi}{2}) \cup (\pi, \frac{3\pi}{2})$

(5) $f(x) = xe^x$

$$f'(x) = xe^x + e^x \cdot 1 = e^x(\underline{x+1}) = 0$$

$$x = -1$$

$f'(x)$: $-$ | $+$ at -1

f is incr on $(-1, \infty)$

min at $x = -1$

(6) $f(x) = \dfrac{e^x}{x}$

$$f'(x) = \frac{xe^x - e^x}{x^2} = \frac{e^x(\underline{x-1})}{x^2}$$

$f'(x)$: $-$ | $-$ | $+$ at 0 and 1

f is incr on $(1, \infty)$

min at $x = 1$

(18) $f(x) = \dfrac{e^x}{x}$

$$f'(x) = \frac{e^x(x-1)}{x^2}$$

$$f''(x) = \frac{e^x x^2\left[\cancel{e^x}^{\,1} + (x-1)\cancel{e^x}\right] - e^x(x-1)\,2x}{x^4}$$

$$= \frac{xe^x\left[x^2 - 2(x-1)\right]}{x^4}$$

$$= \frac{\cancel{x}e^x\left(x^2 - 2x + 2\right)}{\cancel{x^4}\,x^3} \qquad \Delta = 4 - 8$$

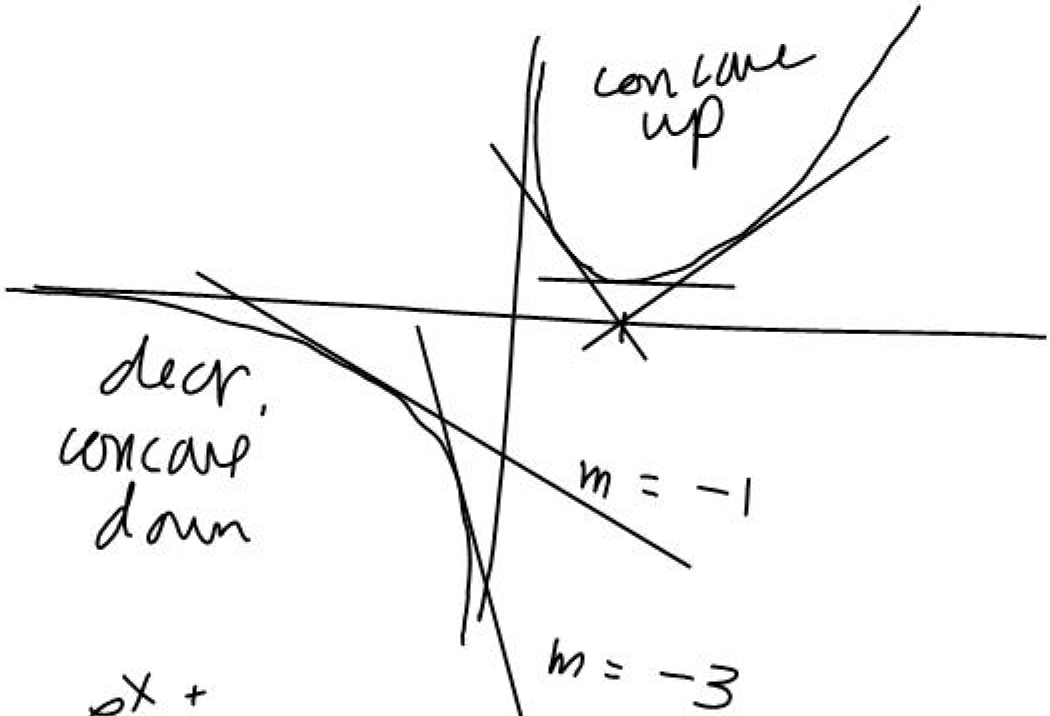
hypercritical values: $x = 0$

$f''(x)$: $\quad - \quad | \quad +$
$\qquad\qquad\quad 0$

f is concave up on $(0, \infty)$.

no flex point

Sketch $f(x) = \frac{e^x}{x}$



$$\lim_{x \to -\infty} \frac{e^{x}\,{}^{+}}{x\,{}^{-}} = 0$$

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(7) $f(x) = \frac{x^2+1}{x+1}$

$$f'(x) = \frac{(x+1)(2x) - (x^2+1)(1)}{(x+1)^2}$$

$$= \frac{x^2+2x-1}{(x+1)^2}$$

C.V. $x = -1$, $x = \frac{-2 \pm \sqrt{\cancel{8}}^{\,2\sqrt{2}}}{2} = -1 \pm \sqrt{2}$

≈ 0.4 or -2.4

$f'(x)$: $+$ | $-$ | $-$ | $+$ at $-1-\sqrt{2}$, -1, $-1+\sqrt{2}$

incr on $(-\infty, -1-\sqrt{2}) \cup (-1+\sqrt{2}, \infty)$

max at $x = -1-\sqrt{2}$

min at $x = -1+\sqrt{2}$

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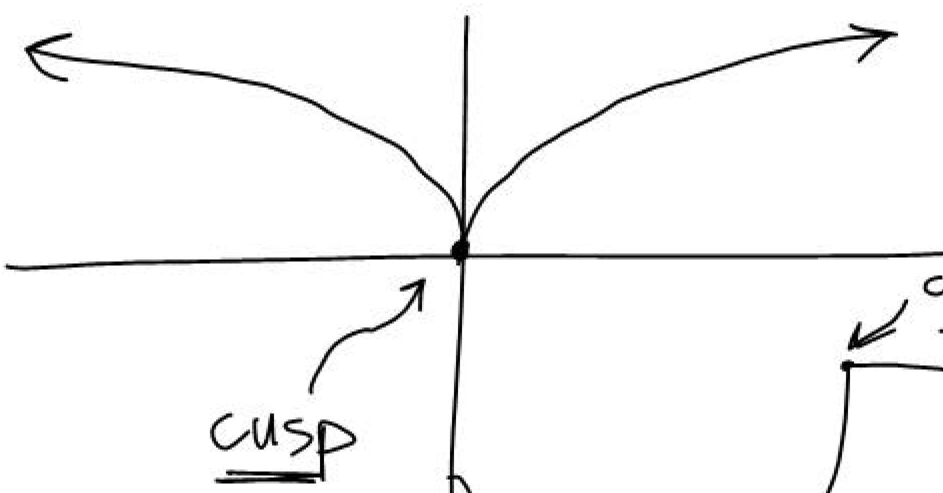
(10) (19) $f(x) = x^{2/3}$

$$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}}$$

$$f''(x) = -\frac{2}{9}x^{-4/3} = \frac{-2}{9x^{4/3}}$$

$f'(x)$: $-$ decr. | 0 | $+$ incr.

$f''(x)$: $-$ conc. down | 0 | $-$ concave down



(13) $f(x) = x^4 - 24x^2 + 5x - 10$

$f'(x) = 4x^3 - 48x + 5$

$f''(x) = 12x^2 - 48 = 0$

$x^2 = 4$

$x = \pm 2$

$f''(x)$: $+$ | -2 | $-$ | 2 | $+$

f is concave up: $(-\infty, -2) \cup (2, \infty)$

flexpts at $x = \pm 2$

(15) $f(x) = x^2 e^x$

$$f'(x) = x^2 \cdot e^x + e^x \cdot 2x$$

$$f''(x) = \left(x^2 e^x + e^x \cdot 2x\right) + \left(e^x \cdot 2 + 2x\, e^x\right)$$

$$= e^x \left(x^2 + 4x + 2\right)$$

hypercritical values $\quad x = \dfrac{-4 \pm \overset{2\sqrt{2}}{\cancel{\sqrt{8}}}}{2}$

$$= -2 \pm \sqrt{2}$$

$f''(x)$: $\quad + \quad | \quad - \quad | \quad +$

$\qquad\qquad -2-\sqrt{2} \qquad\qquad -2+\sqrt{2}$

HW quiz 11-15

$f(x) = x^3 \cdot e^x$

(a) Where is f increasing?

(b) Find f''.