Ex. $2x^2 - xy + y^2 = 4$

(a) Find the ordered pair(s) where $x = 1$.

(b) Write an equation for a tangent at each of the points.

(c) Determine the concavity at each point.

(a) $2(1)^2 - (1)y + y^2 = 4$

$2 - y + y^2 = 4$

$y^2 - y - 2 = 0$

$(y+1)(y-2) = 0$

$y = -1$ or $y = 2$

$$\boxed{\begin{matrix}(1,-1)\\(1,2)\end{matrix}}$$

(b) $2x^2 - xy + y^2 = 4$

$$4x - xy' - y\cdot 1 + 2y\cdot y' = 0$$

$$2yy' - xy' = y - 4x$$

$$y'(2y - x) = y - 4x$$

$$\boxed{y' = \frac{y-4x}{2y-x}}$$

(b) $(1,2)$

$$y'\Big|_{(1,2)} = \frac{2-4(1)}{2(2)-1} = \frac{-2}{3}$$

$$y - y_0 = m(x - x_0) \leftarrow \text{point-slope form}$$

$$\boxed{y - 2 = \frac{-2}{3}(x-1)}$$

$(1, -1)$

$$y - (-1) = \frac{5}{3}(x-1)$$

$$\boxed{y + 1 = \frac{5}{3}(x-1)}$$

(c) $y' = \dfrac{y-4x}{2y-x}$

$$y'' = \frac{(2y-x)(y'-4) - (y-4x)(2y'-1)}{(2y-x)^2}$$

$$y''\Big|_{(1,2)} = \frac{(2\cdot 2-1)\left(-\frac{2}{3}-4\right) - (2-4(1))\left(2\cdot\frac{-2}{3}-1\right)}{(2(2)-1)^2}$$

$$= \frac{\cancel{3}\cdot\left(\frac{-14}{\cancel{3}}\right) + 2\left(\frac{-7}{3}\right)}{9} = \frac{-42-14}{27}$$

$$= \frac{-56}{27} < 0 \Rightarrow \text{concave down}$$

$$y''\Big|_{(1,-1)} = \frac{(-2-1)\left(\frac{5}{3}-4\right) - (-1-4)\left(2\cdot\frac{5}{3}-1\right)}{(-2-1)^2}$$

$$= \frac{-3\cdot\frac{-7}{3} + 5\cdot\frac{7}{3}}{9}$$

$$= \frac{21+35}{27} = \frac{56}{27} > 0$$

$\Rightarrow$ concave up

(d) Solve the relation for y.

$$2x^2 - xy + y^2 = 4$$

$$y^2 - xy + (2x^2-4) = 0$$

treat y as the variable, treat x as a constant

$$y = \frac{x \pm \sqrt{x^2 - 4(2x^2-4)}}{2}$$

$$\boxed{y = \frac{x \pm \sqrt{16-7x^2}}{2}}$$

Ex. Sketch $x^2 - 2xy + y^2 = 1$

$$2x - 2xy' - 2y + 2yy' = 0$$

$$y' = \frac{2y - 2x}{2y - 2x} = 1$$

At $x = 0$, $y^2 = 1$, so $y = \pm 1$

$(0,1)$ and $(0,-1)$