# Derivatives of all sorts

[1] $y = \dfrac{2x+3}{(3x+2)^3}$ use quotient rule

$$\frac{dy}{dx} = \frac{(3x+2)^3\cdot(2) - (2x+3)\,3(3x+2)^2\cdot 3}{(3x+2)^6}$$

$$= \frac{\cancel{(3x+2)^2}\left[\overbrace{(3x+2)}^{6x+4}(2) - \overbrace{9(2x+3)}^{18x-27}\right]}{(3x+2)^{\cancel{6}\,4}}$$

$$= \frac{-12x-23}{(3x+2)^4}$$

[2] $r = \overbrace{e^{2\theta}}^{\text{first}} \cdot \overbrace{\sin^2(3\theta)}^{\text{second}}$ use product rule

$$\frac{dr}{d\theta} = e^{2\theta}\cdot 2\sin(3\theta)\cdot\cos(3\theta)\cdot 3 + \sin^2(3\theta)\cdot e^{2\theta}\cdot 2$$

$$= 2e^{2\theta}\sin(3\theta)\left[3\cos(3\theta) + \sin(3\theta)\right]$$

[3] $v(t) = t^2 \cdot \ln(2t^2+1)$ product rule

$$\frac{dv}{dt} = a(t) = t^2 \cdot \frac{4t}{2t^2+1} + \ln(2t^2+1) \cdot 2t$$

$$= 2t\left(\frac{2t^2}{2t^2+1} + \ln(2t^2+1)\right)$$

[4] $x(t) = t \cdot \sin^{-1} t + \sqrt{1-t^2}$

$$\frac{dx}{dt} = v(t) = t \cdot \frac{1}{\sqrt{1-t^2}} + \sin^{-1} t + \cancel{\tfrac{1}{2}}(1-t^2)^{-\frac{1}{2}} \cdot -\cancel{2}t$$

$$= \frac{t}{\sqrt{1-t^2}} + \sin^{-1} t - \frac{t}{\sqrt{1-t^2}}$$

$$= \sin^{-1} t$$

Note: When we do antiderivatives,

$$\int \sin^{-1} x \, dx = \underbrace{x \sin^{-1} x + \sqrt{1-x^2}}_{\text{antiderivative of } \sin^{-1} x} + C$$

[5] $f(x) = \underbrace{\overbrace{\tfrac{1}{2}\sin x}^{\text{first}} \cdot \overbrace{\cos x}^{\text{second}}}_{\text{product rule}} + \tfrac{1}{2}x$

$$f'(x) = \frac{1}{2}\sin x \cdot (-\sin x) + \cos x \cdot \left(\frac{1}{2}\cos x\right) + \frac{1}{2}$$

$$= \frac{1}{2}\left(-\sin^2 x + \cos^2 x + 1\right) = \frac{1}{2}\left(\cos 2x + 1\right)$$

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Note $\frac{d}{dt}\left[e^{-t}\right] = e^{-t} \cdot \underbrace{(-1)}_{\text{deriv of } -t} = -e^{-t}$

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[6] (typo) Find $y'(t)$ : $y(t) = \dfrac{e^t - e^{-t}}{e^t + e^{-t}}$

$$y'(t) = \frac{(e^t + e^{-t})(e^t + e^{-t}) - (e^t - e^{-t})(e^t - e^{-t})}{(e^t + e^{-t})^2}$$

Note: $e^t \cdot e^{-t} = e^0 = 1$

$$= \frac{\cancel{e^{2t}} + 2 + \cancel{e^{-2t}} - (\cancel{e^{2t}} - 2 + \cancel{e^{-2t}})}{(e^t + e^{-t})^2}$$

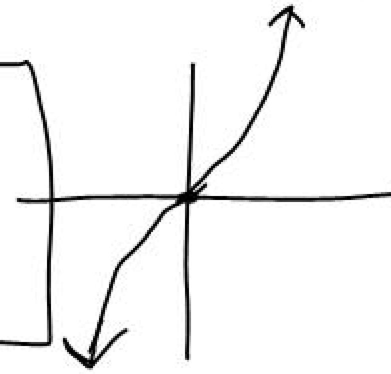
$$= \frac{4}{(e^t + e^{-t})^2} = \operatorname{sech}^2 x$$

(see notes) →

# Hyperbolic Trigonometry Notes

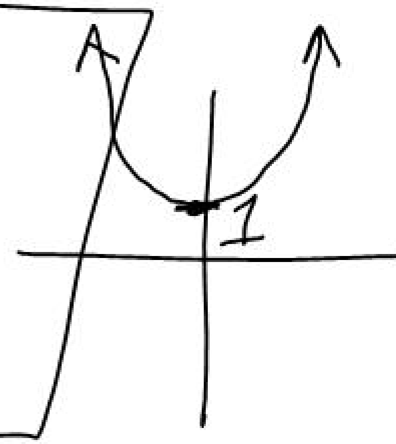
Hyperbolic Sine:

$$\boxed{\sinh x = \frac{e^x - e^{-x}}{2}}$$



Hyperbolic Cosine:

$$\boxed{\cosh x = \frac{e^x + e^{-x}}{2}}$$




$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

[This is #6]

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

$$\boxed{\frac{d}{dx}[\tanh x] = \operatorname{sech}^2 x = \frac{4}{(e^x + e^{-x})^2}}$$

[7] $x\sin^2 y + y\cos^2 x = y$

Use implicit differentiation

$$\left(x \cdot \underbrace{2\sin y \cdot \cos y}_{\text{identity}} \cdot y' + \sin^2 y\right) + \left(y \cdot \underbrace{2\cos x \cdot -\sin x}_{\substack{\text{identity} \\ \text{(take out the neg.)}}} + \cos^2 x \cdot y'\right) = y'$$

Using the identity: $2\sin\theta\cos\theta = \sin 2\theta$

$$\underline{y' \cdot x \cdot \sin 2y} + \sin^2 y - y \cdot \sin 2x + \underline{y' \cdot \cos^2 x} = \underline{y'}$$

move the underlined terms to the left side

$$y'(x \cdot \sin 2y + \underline{\cos^2 x - 1}) = y \cdot \sin 2x - \sin^2 y$$

$$\boxed{y' = \frac{y \cdot \sin 2x - \sin^2 y}{x \cdot \sin 2y - \sin^2 x}}$$

Note: $\sin^2 x + \cos^2 x = 1$

so, $\cos^2 x - 1 = -\sin^2 x$

[8] $\ln y + \underbrace{y \cdot \ln x}_{\text{prod. rule}} = \tan^{-1} x^2$

$$\frac{1}{y} \cdot y' + \left( y \cdot \frac{1}{x} + \ln x \cdot y' \right) = \frac{1}{1+(x^2)^2} \cdot 2x$$

$$y' \left( \frac{1}{y} + \ln x \right) = \frac{2x}{1+x^4} - \frac{y}{x}$$

$$y' = \frac{\frac{(2x)x}{(1+x^4)x} - \frac{y(1+x^4)}{x(1+x^4)}}{\frac{1}{y} + \underline{\frac{\ln x \cdot y}{y}}}$$

get a common denominator on the top and on the bottom

$$= \frac{\frac{2x^2 - y(x^4+1)}{x(x^4+1)}}{\frac{1 + y \ln x}{y}}$$

$$= \frac{2x^2 y - y^2 (x^4+1)}{x(x^4+1)(1+y \cdot \ln x)}$$

[9] $\underbrace{y e^{\sin x}}_{\text{prod. rule}} + \frac{1}{y} = 1$ $\qquad \frac{1}{y} = y^{-1}$

$$\left(y \cdot e^{\sin x} \cdot \cos x + e^{\sin x} \cdot y'\right) - y^{-2} \cdot y' = 0$$

$$y'\left(e^{\sin x} - \frac{1}{y^2}\right) = -y e^{\sin x} \cos x$$

$$y' = \frac{-y e^{\sin x} \cos x}{e^{\sin x} - \frac{1}{y^2}}$$

$$= \frac{y e^{\sin x} \cos x}{\frac{1 - y^2 e^{\sin x}}{y^2}}$$

$$= \frac{y^3 \cdot e^{\sin x} \cdot \cos x}{1 - y^2 \cdot e^{\sin x}}$$

[10] $a(t) = \dfrac{t \sec(t^2)}{e^{t^2}}$

$$j(t) = \frac{da}{dt} = \frac{e^{t^2}\left[t \cdot \sec(t^2) \cdot \tan(t^2) \cdot 2t + \sec(t^2) \cdot 1\right] - t \cdot \sec(t^2) \cdot e^{t^2} \cdot 2t}{e^{2t^2}}$$

$$= \frac{\cancel{e^{t^2}} \sec(t^2)\left[2t^2 \tan(t^2) + 1 - 2t^2\right]}{\cancel{e^{2t^2}}\, e^{t^2}}$$

[11] $x(t) = \ln(\sec t + \tan t)$

$$\frac{dx}{dt} = v(t) = \frac{\sec t \cdot \tan t + \sec^2 t}{\sec t + \tan t}$$

$$= \frac{\sec t\, \cancel{(\tan t + \sec t)}}{\cancel{\sec t + \tan t}}$$

$$= \sec t$$

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<u>Note on antiderivatives</u>: $\int \sec x \, dx = \ln|\sec x + \tan x| + C$

[12] $x^3 - 2x^2y + 3xy^2 - y^3 = x$

$$3x^2 - (\underline{2x^2y'} + y \cdot 4x) + (\underline{3x \cdot 2yy'} + y^2 \cdot 3) - \underline{3y^2y'} = 1$$

$$y'(-2x^2 + 6xy - 3y^2) = 1 - 3x^2 + 4xy - 3y^2$$

$$\boxed{y' = \frac{1 - 3x^2 + 4xy - 3y^2}{-2x^2 + 6xy - 3y^2}}$$