HW quiz 11-2-18

Find $\frac{dy}{dx}$ · $x^3 - x^2y - y^3 = 2$

Derivatives for Logs + Inverse Trig Functions

Find $\frac{dy}{dx}$ for $y = \ln x$

exponentiate: $e^y = e^{\ln x}$

$$e^y = x$$

differentiate implicitly

$$e^y \cdot y' = 1$$

$$y' = \frac{1}{e^y} = \frac{1}{x}$$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

outer ↙ inner ↙

Ex. $\frac{d}{dx}\left[\ln(\cos x)\right]$

$$= \frac{1}{\cos x} \cdot -\sin x$$

$$= -\tan x$$

Ex. $\frac{d}{dx}\left[\ln(x^3)\right] =$

$$a \cdot \ln b = \ln b^a$$

Method 1

$$\frac{1}{x^3} \cdot 3x^2 = \frac{3}{x}$$

Method 2 $\frac{d}{dx}\left[\ln x^3\right] = \frac{d}{dx}\left[3\ln x\right]$

$$= 3 \cdot \frac{1}{x}$$

Ex

$$\frac{d}{dx}\left[x^2 \cdot \ln x\right]$$ Domain: $(0, \infty)$

$$= x^2 \cdot \frac{1}{x} + \ln x \cdot 2x$$

$$= x(1 + 2\ln x)$$

(b) Find the critical values:

$$1 + 2\ln x = 0$$

$$2\ln x = -1$$

$$\ln x = \frac{-1}{2}$$

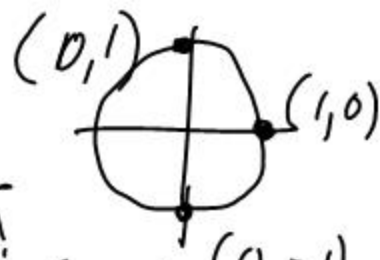
$\log_b a = L$

$b^L = a$

$$x = e^{-1/2} = \frac{1}{\sqrt{e}}$$

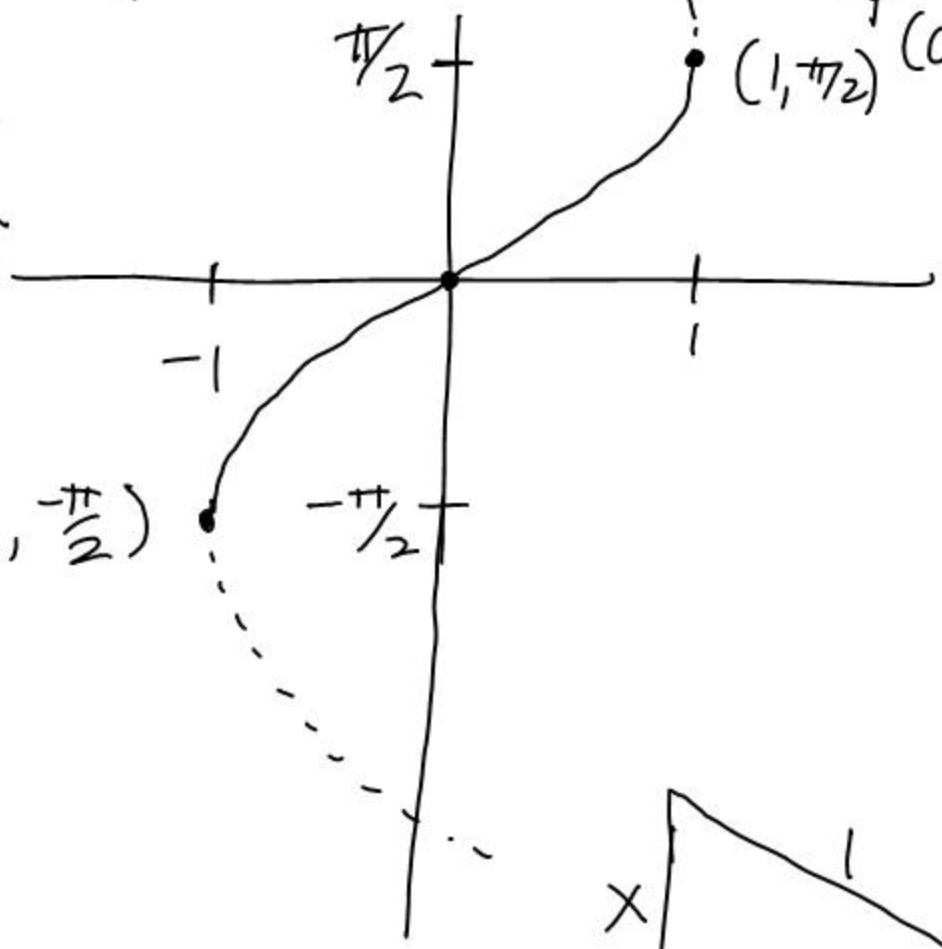
≈ 0.6

Inverse Trig Functions

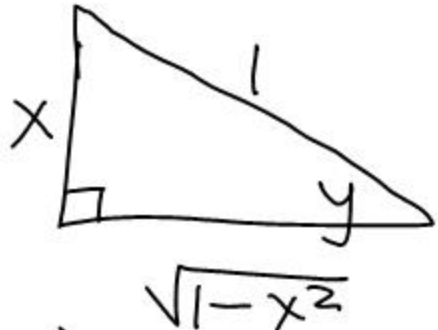


$y = \sin^{-1} x$

$= \arcsin x$



x	$\sin x$
0	0
$\pi/2$	1
$-\pi/2$	-1



Find $\frac{dy}{dx}$: $y = \sin^{-1} x$

$\sin y = \sin(\sin^{-1} x)$

↑ Pythagoras

$\sin y = \frac{x}{1}$

differentiate: $\cos y \cdot y' = 1$

$y' = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$

$$\frac{d}{dx}\left[\sin^{-1} x\right] = \frac{1}{\sqrt{1-x^2}}$$

Ex.

$$\frac{d}{dx}\left[\sin^{-1}\left(\frac{1}{x}\right)\right]$$

Deriv. of inner

$$= \frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}} \cdot \frac{-1}{x^2}$$

$$= \frac{-1}{x^2\sqrt{\frac{x^2-1}{x^2}}} = \frac{-\sqrt{x^2}}{x^2\sqrt{x^2-1}}$$

$$= \frac{-|x|}{x^2\sqrt{x^2-1}} < 0$$

$y = \sqrt{1-x^2}$

$y^2 = 1 - x^2$

$x^2 + y^2 = 1$

Find $\frac{d}{dx}\left[\tan^{-1} x\right]$

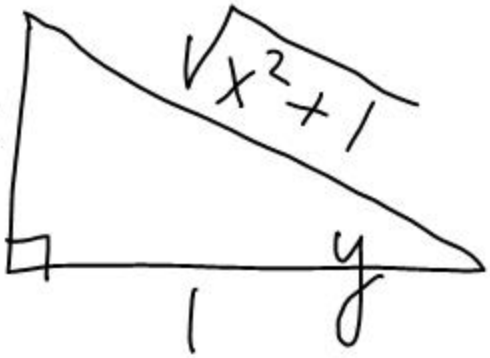
$y = \tan^{-1} x$

$\tan y = x$ ⟩ take the tangent of both sides

$\sec^2 y \cdot y' = 1$ ⟩ differentiate

$$y' = \frac{1}{\sec^2 y} = \frac{1}{x^2+1}$$

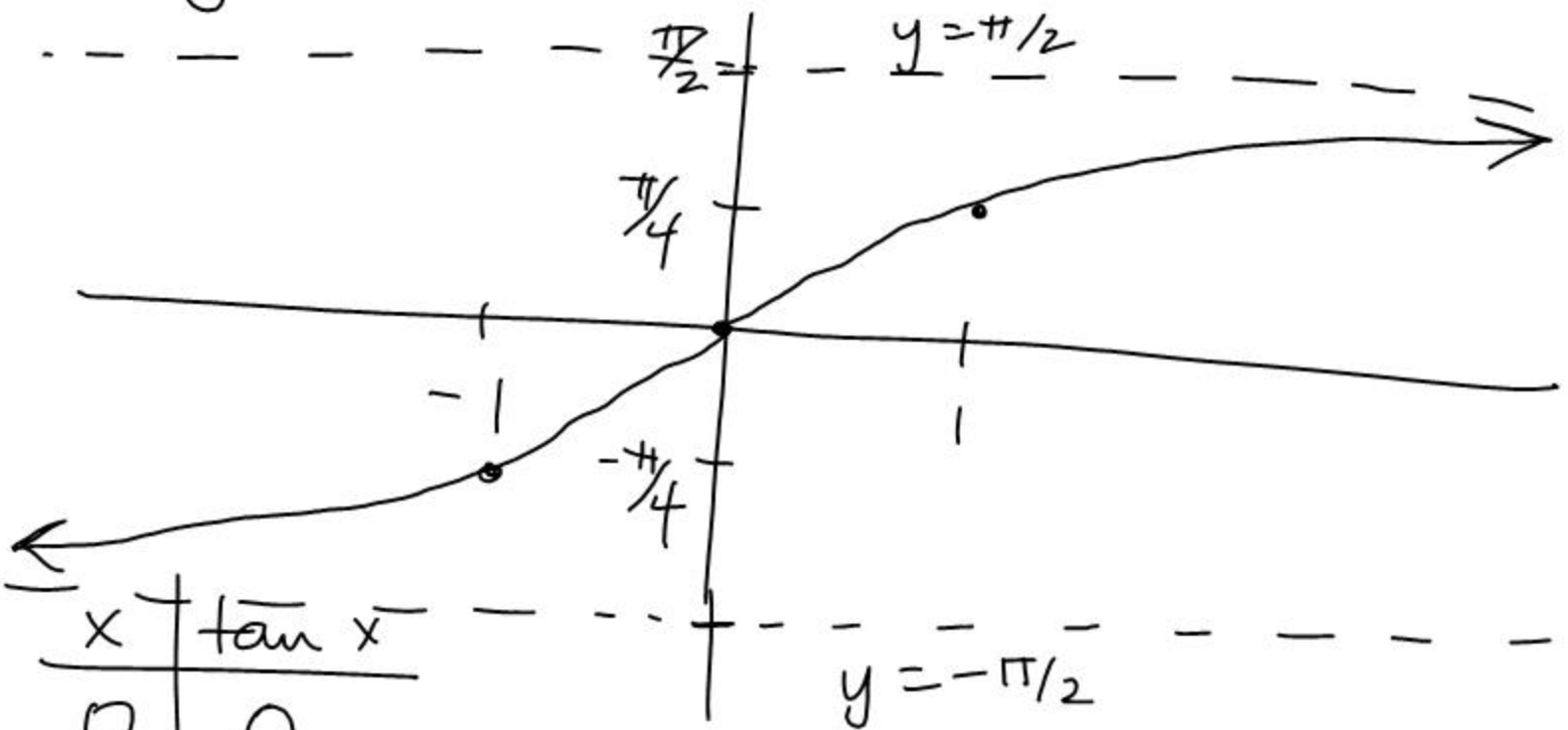
$\tan y = \frac{x}{1}$



$(\sec y)^2 = \left(\frac{\sqrt{x^2+1}}{1}\right)^2$

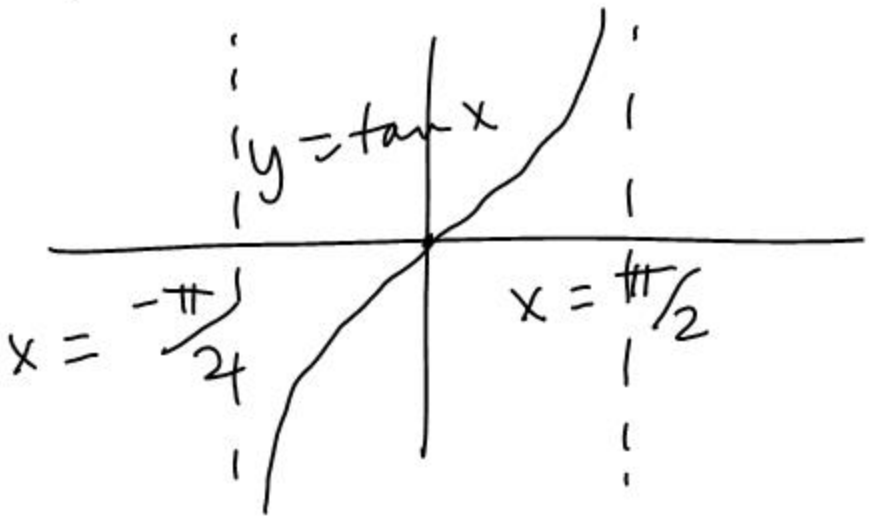
$= x^2+1$

$y = \tan^{-1} x$



x	$\tan x$
0	0
$\pi/4$	1
$-\pi/4$	-1

$$\frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{x^2+1}$$



Ex. $\frac{d}{dx}\left[\tan^{-1} e^x\right]$

$$= \frac{1}{e^{2x}+1} \cdot e^x = \frac{e^x}{e^{2x}+1}$$

Ex. $\frac{d}{dx}\left[\frac{\tan^{-1} x^2}{x}\right]$

$$= \frac{x \cdot \frac{1}{x^4+1} \cdot 2x - \tan^{-1} x^2 \cdot 1}{x^2}$$

$$= \frac{2x^2 - (x^4+1)\cdot \tan^{-1} x^2}{x^2(x^4+1)}$$

HW

Logs + Inverse Trig #1–8, 11–14

$$\frac{d}{dx}\left[\cos^{-1} x\right] = \frac{-1}{\sqrt{1-x^2}}$$

① $y = \ln x^4 = 4 \ln x$

$$\frac{dy}{dx} = 4 \cdot \frac{1}{x} = \frac{4}{x}$$

② $v(t) = t \cdot \ln t$

$$v'(t) = t \cdot \frac{1}{t} + \ln t \cdot 1$$

$$= 1 + \ln t$$

③ $r = \dfrac{\ln \theta}{\theta}$

$$\frac{dr}{d\theta} = \frac{\theta \cdot \frac{1}{\theta} + \ln\theta \cdot 1}{\theta^2} = \frac{1 + \ln\theta}{\theta^2}$$

④ $a(t) = t^2 \cdot \sin^{-1} t$

$$\frac{da}{dt} = t^2 \cdot \frac{1}{\sqrt{1-t^2}} + \sin^{-1} t \cdot 2t$$

$$= t\left[\frac{t}{\sqrt{1-t^2}} + 2\sin^{-1} t\right]$$

(5) $y = \cos^{-1}(2x)$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-4x^2}} \cdot 2 = \frac{-2}{\sqrt{1-4x^2}}$$

(6) $f(s) = \dfrac{\tan^{-1} s}{s^2}$

$$f'(s) = \frac{s^2 \cdot \frac{1}{s^2+1} - \tan^{-1} s \cdot 2s}{s^4 \; s^3}$$

$$= \frac{s - 2(s^2+1)\tan^{-1} s}{s^3(s^2+1)}$$

(7) $a(t) = \sin(\ln t)$

$$\frac{da}{dt} = \cos(\ln t) \cdot \frac{1}{t} = \frac{\cos(\ln t)}{t}$$

(8) $f(x) = (\ln x)^3$

$$f'(x) = 3(\ln x)^2 \cdot \frac{1}{x} = \frac{3(\ln x)^2}{x}$$

(9) $f(x) = x \cdot 2^x$

$$f'(x) = x \cdot 2^x \cdot \ln 2 + 2^x$$

$$= 2^x (x \cdot \ln 2 + 1)$$

(10) $f(x) = 5^{\cos x}$

$$f'(x) = 5^{\cos x} \cdot \ln 5 \cdot -\sin x$$

$$= -5^{\cos x} \cdot \sin x \cdot \ln 5$$

(11) $f(y) = \ln(\csc y - \cot y)$

$$= \frac{-\csc y \cot y + \csc^2 y}{\csc y - \cot y}$$

$$= \frac{\csc y (\csc y - \cot y)}{\csc y - \cot y}$$

$$= \csc y$$

(12) $y = \ln(\cos t)$

$$\frac{dy}{dt} = \frac{1}{\cos t} \cdot -\sin t = -\tan t$$

(13) $f(x) = \tan^{-1}(\ln x)$

$$f'(x) = \frac{1}{(\ln x)^2 + 1} \cdot \frac{1}{x}$$

(14) $f(y) = \dfrac{1}{\ln y}$

$$f'(y) = \frac{0 - \frac{1}{y}}{(\ln y)^2} = \frac{-1}{y(\ln y)^2}$$