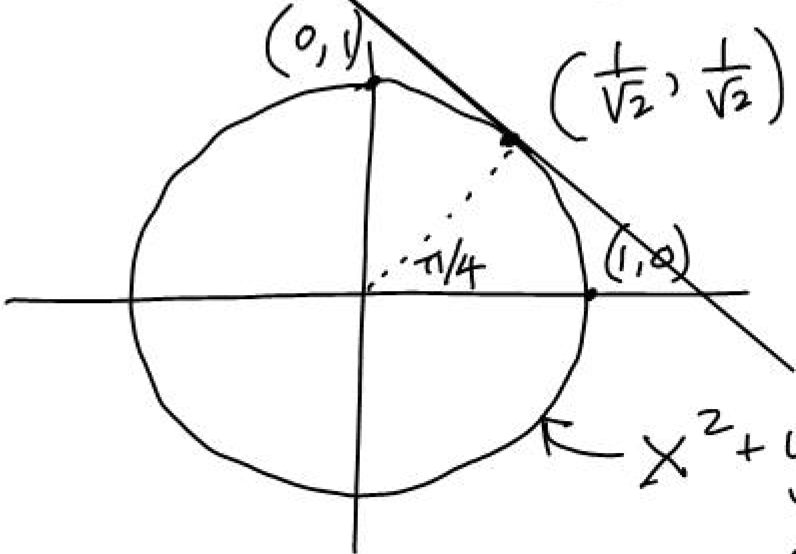
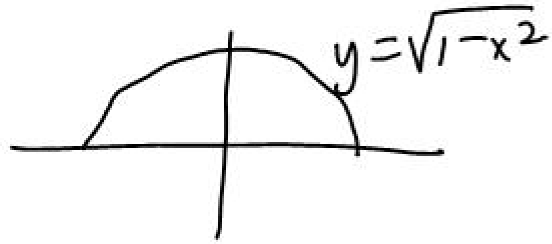
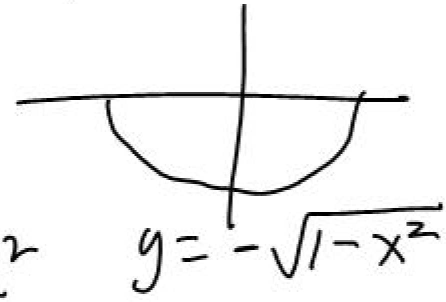
Another tangent line Problem







$$x^2+y^2=1$$

$$y^2 = 1-x^2$$

$$y \pm \sqrt{1-x^2}$$

Find $\left.\dfrac{dy}{dx}\right|_{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)}$

$$y = (1-x^2)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{\cancel{2}}(1-x^2)^{-1/2}\cdot(-\cancel{2}x)$$

$$= \frac{-x}{\sqrt{1-x^2}}$$

$$\left.\frac{dy}{dx}\right|_{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)} = \frac{-\frac{1}{\sqrt{2}}}{\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2}} = \frac{-\frac{1}{\sqrt{2}}}{\sqrt{\frac{1}{2}}} = -1$$

implicit differentiation

Find $\frac{dy}{dx}$ <u>without</u> solving for y first.

$$x^2 + y^2 = 1$$

$$\frac{d}{dx}\left[x^2 + y^2\right] = \frac{d}{dx}[1]$$

$$\frac{d}{dx}\left[x^2\right] + \frac{d}{dx}\left[y^2\right] = \frac{d}{dx}[1]$$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

$$\cancel{2}y\frac{dy}{dx} = -\cancel{2}x$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

$$\left.\frac{dy}{dx}\right|_{\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)} = \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = -1$$

Ex. Find $\frac{dy}{dx}$ for $x^2 + 3xy - y^2 = 1$

$$\frac{d}{dx}\left[x^2\right] + \frac{d}{dx}\left[3xy\right] + \frac{d}{dx}\left[-y^2\right] = \frac{d}{dx}\left[1\right]$$

$$2x + \left(\underbrace{3x \cdot \frac{dy}{dx}} + y \cdot 3\right) \underbrace{- 2y \cdot \frac{dy}{dx}} = 0$$

$$\frac{\cancel{(3x-2y)}\frac{dy}{dx}}{\cancel{3x-2y}} = \frac{-2x - 3y}{3x-2y}$$

$$\boxed{\frac{dy}{dx} = \frac{-2x-3y}{3x-2y}}$$

$(1,3)$ $\quad 1^2 + 3(1)(3) - 3^2 = 1$ ✓

$$\left.\frac{dy}{dx}\right|_{(1,3)} = \frac{-2(1) - 3(3)}{3(1) - 2(3)} = \frac{-11}{-3} = \frac{11}{3}$$

$$x^2 + 3xy - y^2 = 1 \quad \text{hyperbola}$$

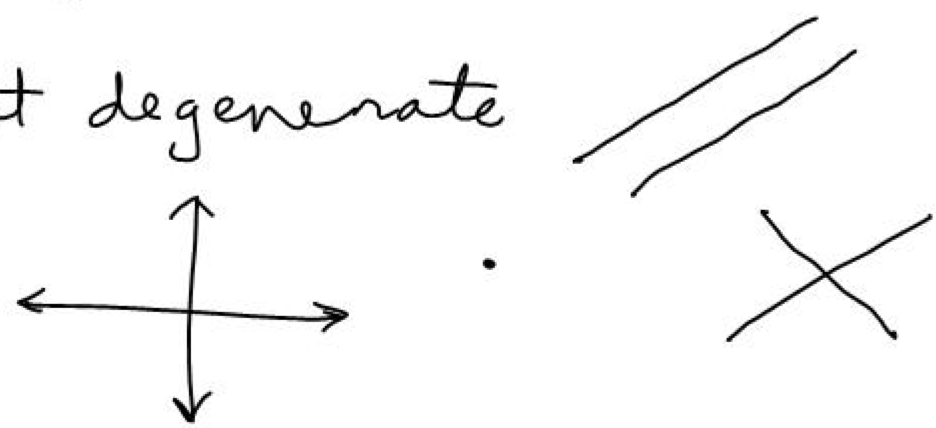
$$B^2 - 4AC = 3^2 - 4(1)(-1) = 13 > 0$$

Every conic section can be written in this form:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

$$B^2 - 4AC \begin{cases} > 0 & \text{hyperbola}^* \\ = 0 & \text{parabola}^* \\ < 0 & \text{ellipse}^* \end{cases}$$

* <u>if</u> it is not degenerate



Find $\frac{dy}{dx}$.

(a) $x^2 \cdot e^y = \sin(x+y)$

$$x^2 \cdot e^y y' + e^y \cdot 2x = \cos(x+y)[1+y']$$

$$\underline{x^2 e^y y'} + 2xe^y = \cos(x+y) \underline{+y'\cos(x+y)}$$

$$[x^2 e^y - \cos(x+y)]y' = \cos(x+y) - 2xe^y$$

$$\boxed{y' = \frac{\cos(x+y) - 2xe^y}{x^2 e^y - \cos(x+y)}}$$

$\boxed{\#2}$

$$x^2 + y^2 = \frac{x+y}{x-y}$$

$$2x + 2yy' = \frac{(x-y)(1+y') - (x+y)(1-y')}{(x-y)^2}$$

$$2x + 2yy' = \frac{\cancel{x} + xy' - y - \cancel{yy'} - \cancel{x} + xy' - y + \cancel{yy'}}{(x-y)^2}$$

$$\frac{-2xy'}{(x-y)^2} + 2yy' = \frac{-2y}{(x-y)^2} - 2x$$

$$y' = \frac{\left(\frac{-2y}{(x-y)^2} - 2x\right)(x-y)^2}{\left(\frac{-2x}{(x-y)^2} + 2y\right)(x-y)^2}$$

$$\boxed{y' = \frac{-2y - 2x(x-y)^2}{-2x + 2y(x-y)^2}}$$

$$\boxed{y' = \frac{y + x(x-y)^2}{x - y(x-y)^2}}$$

$\boxed{\#7}$ (a) $(-1)^2 - 4(1)(1) < 0 \Rightarrow$ ellipse

(b) $2x - (xy' + y) + 2yy' = 0$

$2x - xy' - y + 2yy' = 0$

$-xy' + 2yy' = y - 2x$

$y'(-x+2y)$

$$\boxed{y' = \frac{y - 2x}{-x + 2y}}$$

(c)

(c) $y' = \dfrac{y-2x}{-x+2y} = 0$

$$\boxed{y = 2x}$$

$$\boxed{x^2 - x \cdot y + y^2 = 3}$$

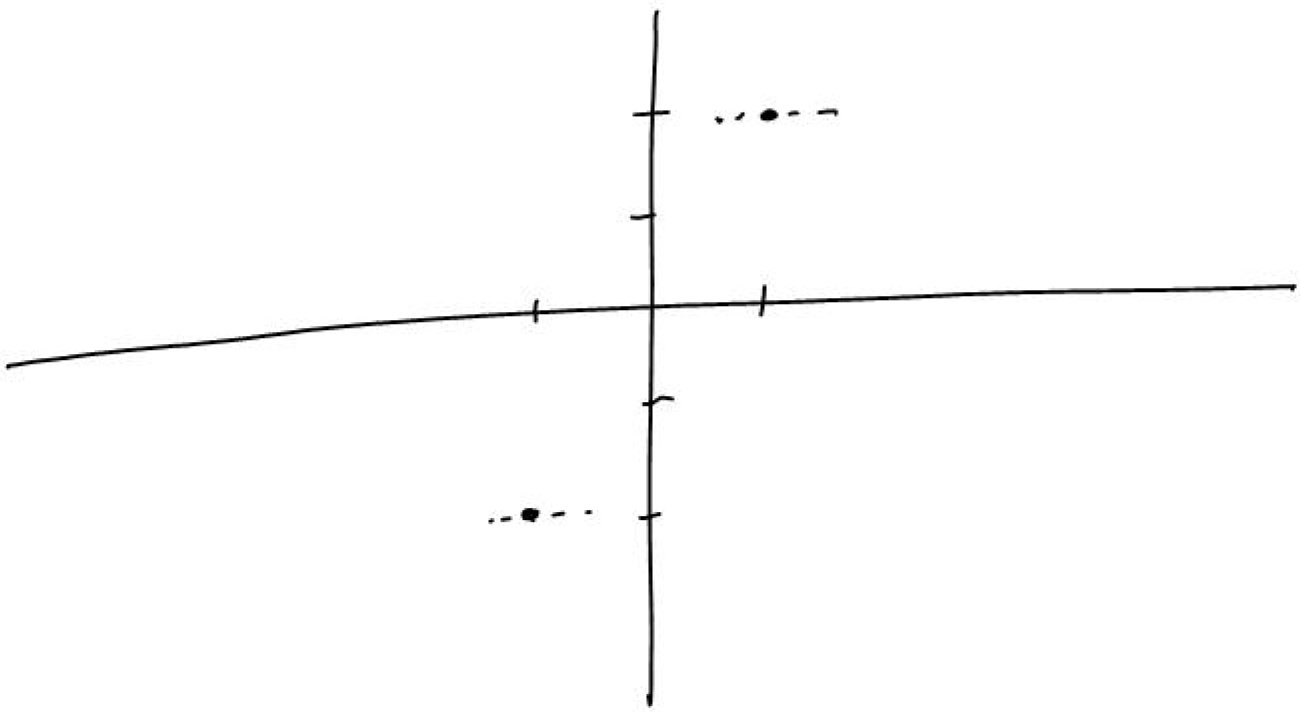
$$x^2 - x(2x) + (2x)^2 = 3$$

$$\cancel{3}x^2 = \cancel{3}\,1$$

$$\boxed{x = \pm 1}$$

(1, 2)

(-1, -2)



Implicit Derivatives

(1) $x^2 - 5xy + 2y^2 = 1$ $\left(\begin{array}{l} 25 - 4(1)(2) > 0 \\ \Rightarrow \text{hyperbola} \end{array} \right)$

$$2x - 5xy' - 5y + 4yy' = 0$$

$$\boxed{y' = \frac{5y - 2x}{4y - 5x}}$$

$\boxed{\#2}$

$$x^2 + y^2 = \frac{x+y}{x-y}$$

$$2x + 2yy' = \frac{(x-y)(1+y') - (x+y)(1-y')}{(x-y)^2}$$

$$2x + 2yy' = \frac{\cancel{x} + xy' - y - \cancel{yy'} - \cancel{x} + xy' - y + \cancel{yy'}}{(x-y)^2}$$

$$\frac{-2xy'}{(x-y)^2} + 2yy' = \frac{-2y}{(x-y)^2} - 2x$$

$$y' = \frac{\left(\frac{-2y}{(x-y)^2} - 2x\right)(x-y)^2}{\left(\frac{-2x}{(x-y)^2} + 2y\right)(x-y)^2}$$

$$\boxed{y' = \frac{-2y - 2x(x-y)^2}{-2x + 2y(x-y)^2}}$$

(3) $x \cdot \sin y + y \cdot \cos x = y^2$

$x \cdot \cos y \cdot y' + \sin y + y(-\sin x) + \cos x \cdot y' = 2yy'$

$xy'\cos y + y'\cos x - 2yy' = -\sin y + y\sin x$

$$\boxed{y' = \frac{y \cdot \sin x - \sin y}{x \cdot \cos y + \cos x - 2y}}$$

(4) $e^y - xy = xy^2$

$e^y \cdot y' - xy' - y = x \cdot 2yy' + y^2$

$y'(e^y - x - 2xy) = y^2 + y$

$$\boxed{y' = \frac{y(y+1)}{e^y - x - 2xy}}$$

(5) $\sin(x+y) - x^2y^2 = x$

$$\cos(x+y)[1+y'] - x^2 \cdot 2yy' - y^2 \cdot 2x = 1$$

$$\cos(x+y) + y' \cdot \cos(x+y) - 2x^2yy' - 2xy^2 = 1$$

$$\boxed{y' = \frac{1+2xy^2 - \cos(x+y)}{\cos(x+y) - 2x^2y}}$$

(6) $x^3 + x^2y - xy^3 + y^4 = 1$

$$3x^2 + \underline{x^2 \cdot y'} + y \cdot 2x \underline{- x \cdot 3y^2y'} - y^3 + \underline{4y^3 \cdot y'} = 0$$

$$\boxed{y' = \frac{y^3 - 2xy - 3x^2}{x^2 - 3xy^2 + 4y^3}}$$

$\boxed{\#7}$ (a) $(-1)^2 - 4(1)(1) < 0 \Rightarrow$ ellipse

(b) $2x - (xy' + y) + 2yy' = 0$

$2x - xy' - y + 2yy' = 0$

$-xy' + 2yy' = y - 2x$

$y'(-x+2y)$

$$\boxed{y' = \frac{y-2x}{-x+2y}}$$

(c) $y' = \dfrac{y-2x}{-x+2y} = 0 \Rightarrow y = 2x$

$x^2 - xy + y^2 = 3$

$x^2 - x(2x) + (2x)^2 = 3$

$3x^2 = 3$

$x^2 = 1$

$\boxed{x = \pm 1}$

$\boxed{\text{Horizontal Tangents at } (1, 2) \text{ and } (-1, -2)}$

(d) $y' = \frac{y-2x}{-x+2y}$ dne $\Rightarrow x = 2y$

$x^2 - xy + y^2 = 3$

$(2y)^2 - (2y)y + y^2 = 3$

$3y^2 = 3$

$y = \pm 1$

$\boxed{\text{Vertical Tangents at } (2,1) \text{ and } (-2,-1)}$

(e) $y' = \frac{y-2x}{-x+2y} = 1$

$y - 2x = -x + 2y$

$-x = y$

$x^2 - xy + y^2 = 3$

$x^2 - x(-x) + (-x)^2 = 3$

$3x^2 = 3$

$x = \pm 1$

$\boxed{\text{Tangent lines with } m = 1 \text{ at } (1,-1) \text{ and } (-1,1)}$

(f) $y' = \frac{y-2x}{-x+2y} = -1$

$y - 2x = x - 2y$

$3y = 3x$

$x = y$

$x^2 - x(x) + x^2 = 3$

$x = \pm\sqrt{3}$

$x \approx \pm 1.7$

$\boxed{\text{Tangents } m = -1 \text{ at } (\sqrt{3}, \sqrt{3}) \text{ and } (-\sqrt{3}, -\sqrt{3})}$

X-intercepts

$x^2 - x(0) + (0)^2 = 3$

$x = \pm\sqrt{3} \approx 1.7$

y-intercepts

$0^2 - 0(y) + y^2 = 3$

$y = \pm\sqrt{3}$