# Derivatives to Know & Love

| $f(x)$ | $f'(x)$ |
|---|---|
| $\sin x$ | $\cos x$ |
| $\cos x$ | $-\sin x$ |
| $\tan x$ | $\sec^2 x$ |
| $\cot x$ | $-\csc^2 x$ |
| $\sec x$ | $\sec x \cdot \tan x$ |
| $\csc x$ | $-\csc x \cdot \cot x$ |
| $e^x$ | $e^x$ |
| $\sin^{-1} x$ | ________ |
| $\cos^{-1} x$ | ________ |
| $\tan^{-1} x$ | ________ |
| $\ln x$ | ________ |

(h) $$\frac{\cos x - x(-\sin x)}{\cos^2 x} = \frac{\cos x + x \cdot \sin x}{\cos^2 x}$$

(i)
$$f'(x) = \frac{e^x(x \cdot -\sin x + \cos x) - (x \cos x) \cdot e^x}{e^{2x}}$$

$$= \frac{\cancel{e^x}(\cos x - x\sin x - x\cos x)}{\cancel{e^{2x}}\, e^x}$$

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Ex. $\frac{d}{dx}\left[\frac{3x^2+4}{4x^2-1}\right]$

$$\frac{(4x^2-1)(6x) - (3x^2+4)(8x)}{(4x^2-1)^2}$$

$$\frac{\cancel{24x^3} - 6x - \cancel{24x^3} - 32x}{(4x^2-1)^2} = \frac{-38x}{(4x^2-1)^2}$$

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# Composite Functions

$f(x) = 2x - 3$

$g(x) = 4x^2 + 2x - 1$

composite functions

$f(g(x)) = 2(4x^2 + 2x - 1) - 3$

$g(f(x)) = 4(2x-3)^2 + 2(2x-3) - 1$

# The Chain Rule

$$\frac{d}{dx}\left[f(g(x))\right] = \underbrace{f'(\underline{g(x)})} \cdot g'(x)$$

outer function

inner function

outer function

Ex. $\frac{d}{dx}\left[(\underbrace{5x+4}_{\text{inner function}})^8\right]$

$= 8(5x+4)^7 \cdot 5$

$= 40(5x+4)^7$

Ex. $\frac{d}{dx}\left[\sqrt{x^2+1}\right]$

$= \cancel{\tfrac{1}{2}}(x^2+1)^{-1/2} \cdot \cancel{2}x$

$= \dfrac{x}{\sqrt{x^2+1}}$

Ex. $\frac{d}{dx}\left[ e^{4x^3} \right]$

inner : $4x^3$
outer : $e^x$

$= \frac{d}{dx}\left[ \exp(4x^3) \right]$

$f(x) = e^x$
$g(x) = 4x^3$
$f(g(x)) = e^{g(x)}$

$= e^{4x^3} \cdot 12x^2$

$\sin(4x^3)$

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Ex. $\frac{d}{dx}\left[ \sin x^2 \right]$

$\sin(x^2)$

inner

outer : $\sin x$

$= \cos x^2 \cdot 2x$

$= \underbrace{2x} \cdot \underline{\cos x^2}$

$\cos x$

$\sqrt{\phantom{x}x}$

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$$f'(x) = \frac{(x+1)^3 \cdot e^{2x} \cdot 2 - e^{2x} \cdot 3(x+1)^2 \cdot 1}{(x+1)^6}$$

$$= \frac{e^{2x} \cancel{(x+1)^2} \left[ (x+1)(2) - 3 \right]}{(x+1)^{\cancel{6}\, 4}}$$

$$= \frac{e^{2x}(2x-1)}{(x+1)^4}$$

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HW Quotient Rule #3 j k l

Chain Rule #3, 7, 11, 15, 19

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Tues 10/23 finish chain rule

Thurs 10/25 "the big 5"

Mon 10/~~30~~ 29 Test on short cuts