① $f(x) = 3x^4$ $\quad x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$

$$f'(x) = \lim_{h \to 0} \frac{3(x+h)^4 - 3x^4}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^4} + 12x^3h + 18x^2h^2 + 12xh^3 + 3h^4 \cancel{-3x^4}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(12x^3 + \cancel{18x^2h} + \cancel{12xh^2} + \cancel{3h^3})}{\cancel{h}} = 12x^3$$

Derivative Shortcuts

① Power Rule $\quad \frac{d}{dx}[x^n] = n \cdot x^{n-1}$

② Constant Rule $\quad \frac{d}{dx}[c] = 0$

③ Constant Multiple Rule

$$\frac{d}{dx}[c \cdot f(x)] = c \cdot f'(x$$

④ Sum Rule $\quad \frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

3 Special Limits

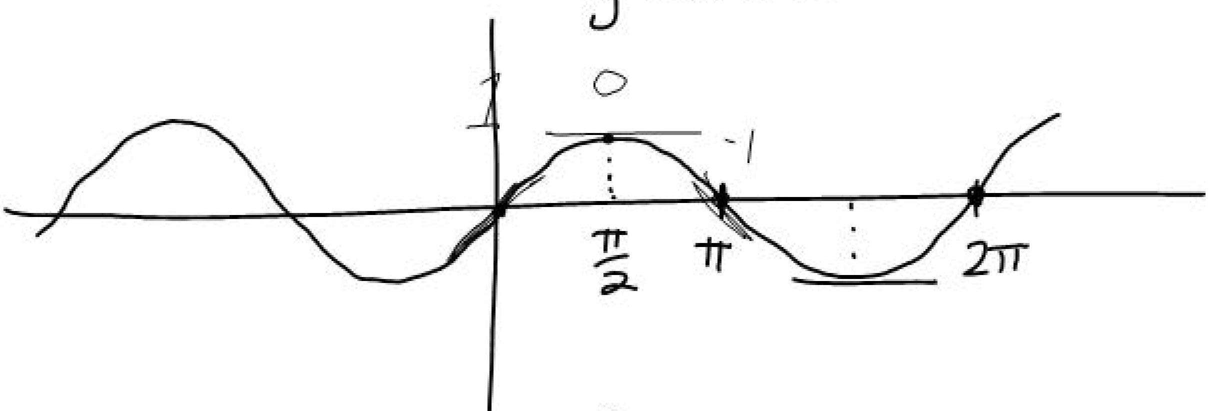
① $\lim_{h \to 0} \frac{\sin h}{h} = 1$

② $\lim_{h \to 0} \frac{\cos h - 1}{h} = 0$

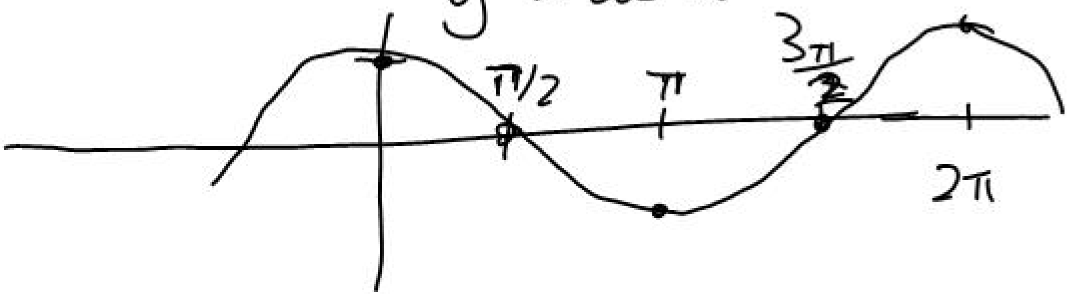
③ $\lim_{h \to 0} \frac{e^h - 1}{h} = 1$

Find $f'(x)$ for $f(x) = \sin x$

$y = \sin x$



$y' = \cos x$



Prove: $\frac{d}{dx}[\sin x] = \cos x$ ✓

$$f'(x) = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cosh + \cos x \sinh - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \underbrace{(\cosh - 1)}_{\to 0}}{h} + \lim_{h\to 0} \frac{\cos x \overbrace{\sinh}^{\to 1}}{h}$$

$$= \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\cos(x+h) = \cos x \cosh - \sin x \sinh$$

Prove: $\frac{d}{dx}\left[e^x\right] = e^x$

The natural exponential function

$$f'(x) = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$$a^m \cdot a^n = a^{m+n}$$

$$= \lim_{h \to 0} \frac{e^x \cdot e^h - e^x}{h}$$

$$= \lim_{h \to 0} \frac{e^x \overbrace{(e^h - 1)}^{\to 1}}{h} = e^x$$

The Product Rule

$$\boxed{\frac{d}{dx}\left[u \cdot v\right] = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx} \quad \text{or } u \cdot v' + v \cdot u'}$$

$1^{st} \cdot$ Deriv of 2^{nd} + $2^{nd} \cdot$ Deriv of 1^{st}

#2f $f(x) = \sin x \cdot \cos x = \frac{1}{2}\sin 2x$

$$f'(x) = \sin x \cdot (-\sin x) + \cos x \cdot (\cos x)$$

$$= \cos^2 x - \sin^2 x$$

$$= \cos 2x$$

HW prove: $\frac{d}{dx}[\cos x] = -\sin x$

power rule: # 1-22 (repeat)

product rule # 2 a-e