$$(x+h)^5$$

$$= x^5 + \binom{5}{4}x^4h + \binom{5}{3}x^3h^2 + \binom{5}{2}x^2h^3$$

$$+ \binom{5}{1}xh^4 + h^5$$

If $f(x) = x^5$

then $f'(x) = \lim_{h \to 0} \dfrac{(x+h)^5 - x^5}{h}$

$$= \lim_{h \to 0} \frac{5x^4\cancel{h} + 10x^3h^{\cancel{2}} + 10x^2h^{\cancel{3}\,2} + 5xh^{\cancel{4}\,3} + h^{\cancel{5}\,4}}{\cancel{h}}$$

$$= 5x^4$$

Find $f'(x)$ for $f(x) = x^n$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{\left[\cancel{x^n} + \binom{n}{1} x^{n-1} \cdot h + \binom{n}{2} x^{n-2} \cdot h^2 + \cdots + h^n\right] - \cancel{x^n}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left[n x^{n-1} + \cancel{\binom{n}{2} x^{n-2} h} + \cdots + \cancel{h^{n-1}}\right]}{\cancel{h}}$$

$$= n \cdot x^{n-1}$$

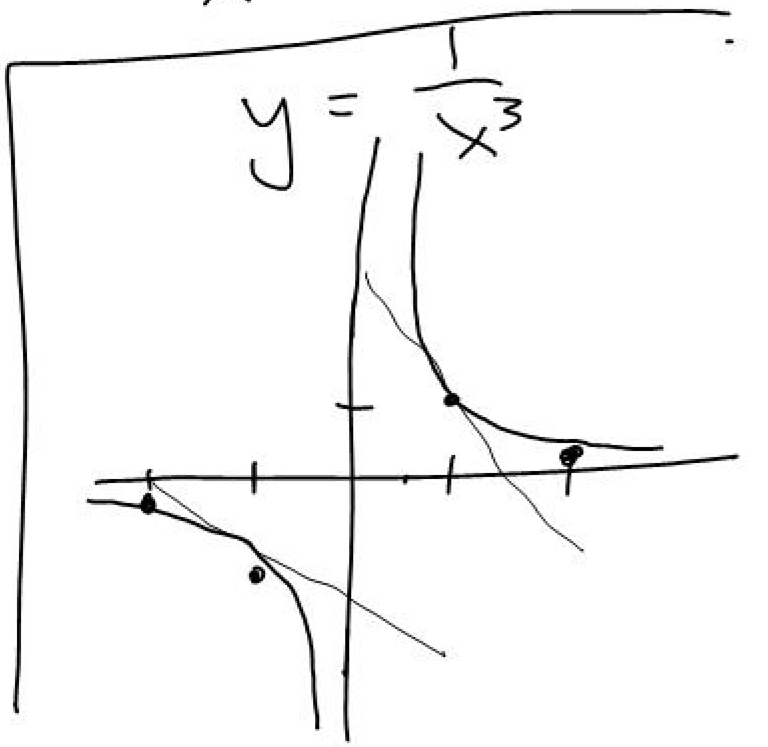
The Power Rule

$$\boxed{\begin{aligned} f(x) &= x^n \\ f'(x) &= n \cdot x^{n-1} \end{aligned}}$$

Ex. $\frac{d}{dx}\left[\frac{1}{x^3}\right] = \frac{d}{dx}\left[x^{-3}\right]$

$$= -3x^{-4} = \frac{-3}{x^4} < 0$$

↑ power rule



Ex. $\frac{d}{dx}\left[\sqrt[3]{x}\right] = \frac{d}{dx}\left[x^{1/3}\right] = \frac{1}{3}x^{-2/3}$

$$= \frac{1}{3x^{2/3}}$$

Ex.

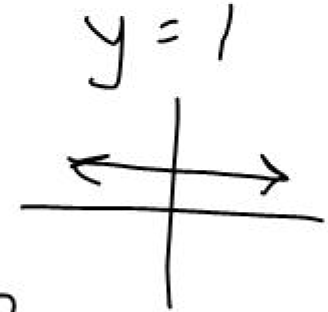
$$\frac{d}{dx}\left[\frac{x^2+x+1}{x}\right]$$

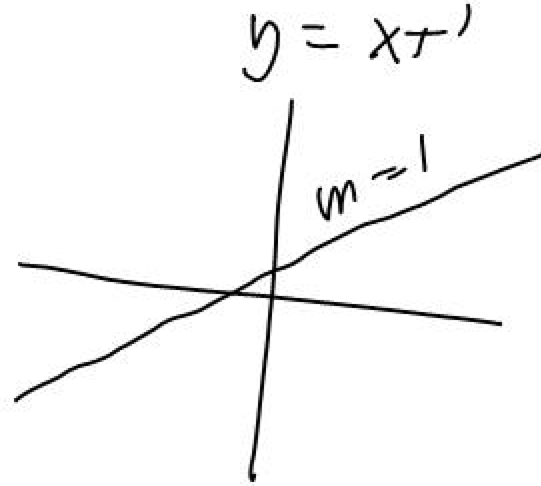
$$= \frac{d}{dx}\left[\frac{x^2}{x}+\frac{x}{x}+\frac{1}{x}\right]$$

$$= \frac{d}{dx}\left[\underbrace{x+1}+x^{-1}\right]$$

$$= 1 + 0 + -x^{-2}$$

$$= 1 - \frac{1}{x^2}$$





Power rule

#21. $f(x) = \dfrac{1}{2\sqrt{x}} - \dfrac{2}{x^3}$

$= \frac{1}{2} x^{-1/2} - 2x^{-3}$

$f'(x) = -\frac{1}{4} x^{-3/2} + 6x^{-4}$

$= \dfrac{-1}{4x^{3/2}} + \dfrac{6}{x^4}$

The Constant Rule

$$\frac{d}{dx}[\text{constant}] = 0$$

Constant Multiple Rule

$$\frac{d}{dx}[c \cdot f(x)] = c \cdot f'(x)$$

Ex. $f(x) = 4x^3$

$f'(x) = 4 \cdot 3x^2 = 12x^2$

The Sum Rule

$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x)$$

Some Special Limits (to memorize)

---

① $$\lim_{h \to 0} \frac{\sin h}{h} = 1$$

② $$\lim_{h \to 0} \frac{\cos h - 1}{h} = 0$$

③ $$\lim_{h \to 0} \frac{e^h - 1}{h} = 1$$
